On stable convex sets

Abstract: A convex set is stable if the midpoint map $(x,y)\mapsto\frac{1}{2}(x+y)$ is open. After reporting on the history of the theory, developed in the 1970's and the first decade of the 21st century, the talk addresses applications to quantum information theory, namely continuity of entanglement monotones and of von Neumann entropy for density matrices on a separable Hilbert space. In finite dimensions, the speaker will report on stability in the context of the continuity of inference maps. This has non-trivial intersections with ground state problems, geometry of reduced density matrices, and continuity problems of correlation quantities.

The theory of stable convex sets emerged in the 1970's from the theory of regular Borel probability measures on a compact convex set K. Vesterstr{\o}m and O'Brien showed that the lower envelope of any continuous real function on K is continuous if and only if the barycenter map is open if and only if K is stable. In finite dimensions, Papadopoulou proved that K is stable if and only if the face function of K is lower semi-continuous. In the first decade of the 21st century, Holevo, Shirokov, and Protasov developed a theory of generalized compactness, called μ-compactness, and they extended the stability theory to it. In particular, the density matrices on a separable Hilbert space form a stable μ-compact convex set, and applications to entanglement monotones and von Neumann entropy were found.