Rational $SO(3)$-equivariant cohomology theories

supervised by

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Topological spaces occur almost everywhere in mathematics. It has been particularly fruitful to study homotopies of these spaces (and maps between them). Symmetries appear everywhere in mathematics, so it is logical to study topological spaces that have symmetries. In turn we should look at homotopies that preserve these symmetries. This area is known as equivariant homotopy theory.

Equivariant homotopy theory is particularly exciting when one considers groups of symmetries like $O(n)$ or $U(n)$. These are groups that are not discrete, they have their own topology and are very well behaved, specifically they are compact Lie groups.

Thus studying equivariant homotopy theory for $G$ a compact Lie group is a worthy and interesting task. A spectacularly successful method has been to look at equivariant cohomology theories (a generalisation of cohomology theories to account for the data of the symmetries).

A long-standing program of Greenlees is to classify the collection of $G$-equivariant cohomology theories for varying $G$. This program has been fully completed in the following cases: finite groups [Bar09], products of $SO(2)$ [GS11], and $O(2)$ [Bar12].

The case of $SO(3)$ has been partially completed in [Gre01]. Recent advances by Greenlees and Shipley in the case of the torus should allow one to make a more complete classification for $G = SO(3)$. This would be the primary task of the project and could be extended to $SU(2)$ if time permits. When the foundations are understood, other directions are possible, such as a study of equivariant $K$-theory for this group, or looking for an equivariant version of Bousfield’s united $K$-theory [Bou90].

The student should have attended courses on algebraic topology and topology. Some algebra will also be useful. This project will require the student to become familiar with the abstract language of model categories [DS95] and modern categories of spectra [MM02].

REFERENCES