Topological Orbit Equivalence: New Developments

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Abstract. In 1959, H. Dye introduced the notion of orbit equivalence and proved that any two ergodic finite measure preserving transformations on a Lebesgue space are orbit equivalent. One year later, he also conjectured that an arbitrary ergodic action of a discrete amenable group is orbit equivalent to a $\mathbb{Z}$-action. This conjecture was proved by Ornstein and Weiss in 1980. The most general case was proved by Connes, Feldman and Weiss by establishing that an amenable non-singular countable equivalence relation $\mathcal{R}$ can be generated by a single transformation, or equivalently, is hyperfinite, i.e., $\mathcal{R}$ is up to a null set a countable increasing union of finite equivalence relations.

For the Borel case, B. Weiss proved that actions of $\mathbb{Z}^n$ are (orbit equivalent to) hyperfinite Borel equivalence relations, whose classification was obtained by Dougherty, Jackson and Kechris.

In my first lecture, I will give an overview of these results in the measurable and the Borel case, before introducing the concept of orbit equivalence in topological dynamics and giving examples of minimal $\mathbb{Z}$-actions on the Cantor set.

In 1995, Giordano, Putnam and Skau proved that minimal $\mathbb{Z}$-actions on the Cantor set were orbit equivalent to approximately finite (AF) relations and their classification was given. Since then some special classes of minimal free actions of $\mathbb{Z}^2$ on the Cantor set were shown to be affable (i.e., orbit equivalent to AF-relations).

In my second talk, after having defined approximately finite (AF) relations and presented their classification, I will indicate the main steps of the proof of the general result obtained in a joint effort with H. Matui, I. Putnam and C. Skau and whose statement is the following:

Theorem. *Any minimal, free $\mathbb{Z}^2$-action on the Cantor set is affable.*