Steel Design to Eurocode 3

Combined axial compression and bending

Uniform members in bending and axial compression demonstrate complex structural behaviour

Interaction Method

When using the interaction method you will need to refer to Clause 6.3.3 of EN 1993-1-1, and you will also need to refer to Annex A or B depending on the specific method being used.

Clause 6.3.3(1)

When checking uniform members in bending and axial compression, a distinction is made for:

- members not susceptible to torsional deformation (e.g. SHS, CHS, fully restrained members)
- members susceptible to torsional deformation

Clause 6.3.3(2)

The resistance of the cross-sections at each end of the member should also satisfy the requirements given in Clause 6.2

Clause 6.3.3(3)

For members of structural systems the resistance check may be carried out on the basis of the individual single span members regarded as cut out of the system.

Second-order effects of the sway system (P-Δ effects) have to be taken into account, whether by the end moments of the member or by means of appropriate buckling lengths respectively.

Clause 6.3.3(4)

Members which are subjected to combined bending and axial compression should satisfy both:

Equation 6.61

$$\frac{N_{Ed}}{\chi_y N_{Rk}/\gamma_{M1}} + k_{yy}\frac{M_{y,Ed}+\Delta M_{y,Ed}}{\chi_{LT} M_{y,Rk}/\gamma_{M1}} + k_{yz}\frac{M_{z,Ed}+\Delta M_{z,Ed}}{M_{z,Rk}/\gamma_{M1}} \leq 1$$

Equation 6.62

$$\frac{N_{Ed}}{\chi_z N_{Rk}/\gamma_{M1}} + k_{zy}\frac{M_{y,Ed}+\Delta M_{y,Ed}}{\chi_{LT} M_{y,Rk}/\gamma_{M1}} + k_{zz}\frac{M_{z,Ed}+\Delta M_{z,Ed}}{M_{z,Rk}/\gamma_{M1}} \leq 1$$

where:

N_{Ed} $M_{y,Ed}$ $M_{z,Ed}$	design values of the compression force and the maximum moments about the y-y and z-z axes along the member, respectively
$\Delta M_{y,Rd}$ $\Delta M_{z,Rk}$	moments due to the shift of the centroidal axis according to 6.2.9.3
X_y X_z	reduction factors due to flexural buckling from clause 6.3.1
X_{LT}	reduction factor due to lateral torsional buckling from clause 6.3.2
k_{yy}, k_{yz}, k_{zy}, k_{zz}	interaction factors k_{ij}.

Table 6.7 – Values for N_{Rk}, $M_{i,Rk}$ and $\Delta M_{i,Ed}$

$N_{Rk} = f_y A_i$ $M_{i,Rk} = f_y W_i$

Class	1	2	3	4
A_I	A	A	A	A_{eff}
W_y	$W_{pl,y}$	$W_{pl,y}$	$W_{el,y}$	$W_{eff,y}$
W_z	$W_{pl,z}$	$W_{pl,z}$	$W_{el,z}$	$W_{eff,z}$
$\Delta M_{Y,Ed}$	0	0	0	$e_{N,y}N_{Ed}$
$\Delta M_{z,Ed}$	0	0	0	$e_{N,z}N_{Ed}$

NOTE1 : For members not susceptible to torsional deformation χ_{LT} would be $\chi_{LT} = 1.0$

NOTE 2: $N_{b,Rd} = \chi\ N_{Rk}/\gamma_{M1}$

Interaction Factors k_{ij}

Interaction factors are obtained from one of two methods:

- Method 1 (given in Annex A)
- Method 2 (given in Annex B)

Annex A (Method 1)

- Use Table A.1 of EN 1993-1-1

Equivalent uniform moment factors $C_{mi,0}$ depend on the shape of the bending moments diagram and these factors are determined from Table A.2 of EN 1993-1-1

Annex B (Method 2)

- Use Table B.1 of EN 1993-1-1 for members not susceptible to LTB
- Use Table B.2 of EN 1993-1-1for members that are susceptible to LTB.

Determine the equivalent uniform moment factors from Table B.3 of EN 1993-1-1.

Annex A

Table A.1 Interaction factors kij for interaction formula in clause 6.3.3 (4)

Interaction factors	Design assumptions	
	elastic cross-sectional properties class 3, class 4	plastic cross-sectional properties class 1, class 2
k_{yy}	$C_{my} C_{mLT} \frac{\mu_y}{1-\frac{N_{Ed}}{N_{cr,y}}}$	$C_{my} C_{mLT} \frac{\mu_y}{1-\frac{N_{Ed}}{N_{cr,y}}} \frac{1}{C_{yy}}$
k_{yz}	$C_{mz} \frac{\mu_y}{1-\frac{N_{Ed}}{N_{cr,z}}}$	$C_{mz} \frac{\mu_y}{1-\frac{N_{Ed}}{N_{cr,z}}} \frac{1}{C_{yz}} 0.6 \sqrt{\frac{w_z}{w_y}}$
k_{zy}	$C_{my} C_{mLT} \frac{\mu_y}{1-\frac{N_{Ed}}{N_{cr,y}}}$	$C_{my} C_{mLT} \frac{\mu_z}{1-\frac{N_{Ed}}{N_{cr,y}}} \frac{1}{C_{zy}} 0.6 \sqrt{\frac{w_y}{w_z}}$
k_{zz}	$C_{mz} \frac{\mu_y}{1-\frac{N_{Ed}}{N_{cr,z}}}$	$C_{mz} \frac{\mu_z}{1-\frac{N_{Ed}}{N_{cr,z}}} \frac{1}{C_{zz}}$

Auxiliary terms:	
$\mu_y = \frac{1-\frac{N_{Ed}}{N_{cr,y}}}{1-\chi_y \frac{N_{Ed}}{N_{cr,y}}}$ $\mu_z = \frac{1-\frac{N_{Ed}}{N_{cr,z}}}{1-\chi_z \frac{N_{Ed}}{N_{cr,z}}}$ $w_y = \frac{W_{el,y}}{W_{pl,y}} \leq 1.5$ $w_z = \frac{W_{el,z}}{W_{pl,z}} \leq 1.5$ $n_{pl} = \frac{N_{Ed}}{N_{Rk}/\gamma_{M1}}$ C_{my} see Table A.2 $\alpha_{LT} = 1 - \frac{I_T}{I_y} \geq 0$	$C_{yy} = 1 + (w_y - 1)\left[\left(2 - \frac{1.6}{w_y} C_{my}^2 \bar{\lambda}_{max} - \frac{1.6}{w_y} C_{my}^2 \bar{\lambda}_{max}^2\right) n_{pl} - b_{LT}\right] \geq \frac{W_{el,y}}{W_{pl,y}}$ with $b_{LT} = 0.5 \alpha_{LT} \bar{\lambda}_0^2 \frac{M_{y,Ed}}{\chi_{LT} M_{pl,y,Rd}} \frac{M_{z,Ed}}{M_{pl,z,Rd}}$ $C_{yz} = 1 + (w_z - 1)\left[\left(2 - 14 \frac{C_{mz}^2 \bar{\lambda}_{max}^2}{w_z^2}\right) n_{pl} - c_{LT}\right] \geq 0.6 \sqrt{\frac{w_z}{w_y}} \frac{W_{el,z}}{W_{pl,z}}$ with $c_{LT} = 10 \alpha_{LT} \frac{\bar{\lambda}_0^2}{5+\bar{\lambda}_z^4} \frac{M_{y,Ed}}{C_{my} \chi_{LT} M_{pl,y,Rd}}$ $C_{zy} = 1 + (w_y - 1)\left[\left(2 - 14 \frac{C_{my}^2 \bar{\lambda}_{max}^2}{w_y^5}\right) n_{pl} - d_{LT}\right] \geq 0.6 \sqrt{\frac{w_y}{w_z}} \frac{W_{el,y}}{W_{pl,y}}$ with $d_{LT} = 2 \alpha_{LT} \frac{\bar{\lambda}_0}{0.1+\bar{\lambda}_z^4} \frac{M_{y,Ed}}{C_{my} \chi_{LT} M_{pl,y,Rd}} \frac{M_{z,Ed}}{C_{mz} M_{pl,z,Rd}}$ $C_{zz} = 1 + (w_z - 1)\left[\left(2 - \frac{1.6}{w_z} C_{mz}^2 \bar{\lambda}_{max} - \frac{1.6}{w_z} C_{mz}^2 \bar{\lambda}_{max}^2\right) n_{pl} - e_{LT}\right] \geq \frac{W_{el,z}}{W_{pl,z}}$ with $e_{LT} = 1.7 \alpha_{LT} \bar{\lambda}_0^2 \frac{\bar{\lambda}_0}{0.1+\bar{\lambda}_z^4} \frac{M_{y,Ed}}{C_{my} \chi_{LT} M_{pl,y,Rd}}$

$\bar{\lambda} = \max \left\{ \begin{matrix} \bar{\lambda}_y \\ \bar{\lambda}_z \end{matrix} \right.$

$\bar{\lambda}_0$ = non-dimensional slenderness for lateral-torsional buckling die to uniform bending moment i.e. $\psi = 1.0$ in Table A.2

$\bar{\lambda}_{LT}$= non-dimensional slenderness for lateral torsional buckling

For $\bar{\lambda}_0 = 0$:

$C_{my} = C_{my,0}$

$C_{mz} = C_{mz,0}$

$C_{mLT} = 1.0$

For $\bar{\lambda}_0 > 0$:

$$C_{my} = C_{my,0} + (1 - C_{my,0}) \frac{\sqrt{\varepsilon_y} \alpha_{LT}}{1 + \sqrt{\varepsilon_y} \alpha_{LT}}$$

$C_{mz} = C_{mz,0}$

$$C_{mLT} = C_{my}{}^2 \frac{\alpha_{LT}}{\sqrt{\left(1 - \frac{N_{Ed}}{N_{cr,z}}\right)\left(1 - \frac{N_{Ed}}{N_{cr,T}}\right)}}$$

$\varepsilon_y = \frac{M_{y,Ed}}{N_{Ed}} \frac{A}{W_{el,y}}$ for class 1, 2 and 3 cross-sections

$\varepsilon_y = \frac{M_{y,Ed}}{N_{Ed}} \frac{A_{eff}}{W_{eff,y}}$ for class 4 cross-sections

$N_{cr,y}$ = elastic flexural buckling force about the y-y axis

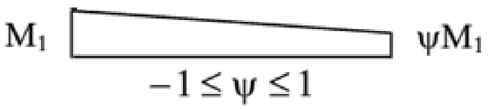
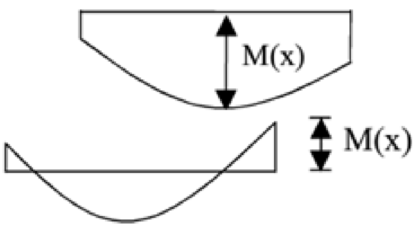
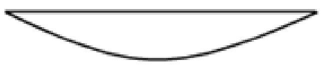
$N_{cr,z}$ = elastic flexural buckling force about the z-z axis

$N_{cr,T}$ = elastic torsional buckling force

I_T = St. Venant torsional constant

I_y = second moment of area about y-y axis

Table A.2 – Equivalent uniform moment factors $C_{mi,0}$

Moment Diagram	Cmi,0
	$C_{mi,0} = 0.79 + 0.21\psi_i + 0.36(\psi_i - 0.33)\frac{N_{Ed}}{N_{cr,i}}$
	$C_{mi,0} = 1 + \left(\frac{\pi^2 E I_i \lvert\delta_x\rvert}{L^2 \lvert M_{i,Ed}(x)\rvert} - 1\right)\frac{N_{Ed}}{N_{cr,i}}$ $M_{i,Ed(x)}$ is the maximum moment $M_{y,Ed}$ or $M_{z,Ed}$ $\lvert\delta_x\rvert$ is the maximum member displacement along the member
 	$C_{mi,0} = 1 - 0.18\frac{N_{Ed}}{N_{cr,i}}$ $C_{mi,0} = 1 + 0.03\frac{N_{Ed}}{N_{cr,i}}$

Annex B

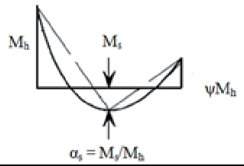
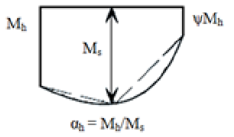
Table B.1 - Interaction factors k_{ij} for members not susceptible to torsional deformations

Interaction factors	Type of sections	Design assumptions	
		elastic cross-sectional properties class 3, class 4	plastic cross-sectional properties class 1, class 2
k_{yy}	I-sections RHS-sections	$C_{my}\left(1+0.6\bar{\lambda}_y\frac{N_{Ed}}{\chi_y N_{Rk}/\gamma_{M1}}\right) \leq C_{my}\left(1+0.6\frac{N_{Ed}}{\chi_y N_{Rk}/\gamma_{M1}}\right)$	$C_{my}\left(1+(\bar{\lambda}_y-0.2)\frac{N_{Ed}}{\chi_y N_{Rk}/\gamma_{M1}}\right) \leq C_{my}\left(1+0.8\frac{N_{Ed}}{\chi_y N_{Rk}/\gamma_{M1}}\right)$
k_{yz}	I-sections RHS-sections	k_{zz}	$0.6k_{zz}$
k_{zy}	I-sections RHS-sections	$0.8k_{yy}$	$0.6k_{yy}$
k_{zz}	I-Sections	$C_{my}\left(1+0.6\bar{\lambda}_z\frac{N_{Ed}}{\chi_z N_{Rk}/\gamma_{M1}}\right) \leq C_{mz}\left(1+0.6\frac{N_{Ed}}{\chi_z N_{Rk}/\gamma_{M1}}\right)$	$C_{mz}\left(1+(2\bar{\lambda}_y-0.6)\frac{N_{Ed}}{\chi_z N_{Rk}/\gamma_{M1}}\right) \leq C_{mz}\left(1+1.4\frac{N_{Ed}}{\chi_z N_{Rk}/\gamma_{M1}}\right)$
	RHS-sections		$C_{mz}\left(1+(\bar{\lambda}_y-0.2)\frac{N_{Ed}}{\chi_z N_{Rk}/\gamma_{M1}}\right) \leq C_{mz}\left(1+0.8\frac{N_{Ed}}{\chi_z N_{Rk}/\gamma_{M1}}\right)$
For I- and H-sections and rectangular hollow sections under axial compression and uniaxial bending $M_{y,Ed}$ the coefficient k_{zy} may be $k_{zy} = 0$			

Table B.2 - Interaction factors k_{ij} for members susceptible to torsional deformations

Interaction factors	Design assumptions	
	elastic cross-sectional properties class 3, class 4	plastic cross-sectional properties class 1, class 2
k_{yy}	k_{yy} from Table B.1	k_{yy} from Table B.1
k_{yz}	k_{yz} from Table B.1	k_{yz} from Table B.1
k_{zy}	$\left[1-\frac{0.05\bar{\lambda}_z}{(C_{mLT}-0.25)}\frac{N_{Ed}}{\chi_z N_{Rk}/\gamma_{M1}}\right] \geq \left[1-\frac{0.05}{(C_{mLT}-0.25)}\frac{N_{Ed}}{\chi_z N_{Rk}/\gamma_{M1}}\right]$	$\left[1-\frac{0.1\bar{\lambda}_z}{(C_{mLT}-0.25)}\frac{N_{Ed}}{\chi_z N_{Rk}/\gamma_{M1}}\right] \geq \left[1-\frac{0.1}{(C_{mLT}-0.25)}\frac{N_{Ed}}{\chi_z N_{Rk}/\gamma_{M1}}\right]$ for $\bar{\lambda}_z < 0.4$: $k_{zy} = 0.6+\bar{\lambda}_z \leq 1-\frac{0.1\bar{\lambda}_z}{(C_{mLT}-0.25)}\frac{N_{Ed}}{\chi_z N_{Rk}/\gamma_{M1}}$
k_{zz}	k_{zz} from Table B.1	k_{zz} from Table B.1

Table B.3 Equivalent uniform factors C_m in Tables B.1 and B.2

Moment Diagram	Range		C_{my} and C_{mz} and C_{mLT} uniform loading	C_{my} and C_{mz} and C_{mLT} concentrated load
	$-1 \le \psi \le 1$		$0.6 + 0.4\psi \ge 0.4$	
	$0 \le \alpha_s \le 1$	$-1 \le \psi \le 1$	$0.2 + 0.8\alpha_s \ge 0.4$	$0.2 + 0.8\alpha_s \ge 0.4$
	$-1 \le \alpha_s \le 0$	$0 \le \psi \le 1$	$0.1 - 0.8\alpha_s \ge 0.4$	$-0.8\alpha_s \ge 0.4$
		$-1 \le \psi \le 0$	$0.1(1-\psi) - 0.8\alpha_s \ge 0.4$	$0.2(-\psi) - 0.8\alpha_s \ge 0.4$
	$0 \le \alpha_h \le 1$	$-1 \le \psi \le 1$	$0.95 + 0.05\alpha_h$	$0.90+0.10\alpha_h$
	$-1 \le \alpha_h \le 0$	$0 \le \psi \le 1$	$0.95 + 0.05\alpha_h$	$0.90 + 0.10\alpha_h$
		$-1 \le \psi \le 0$	$0.95 + 0.05\alpha_h(1+2\psi)$	$0.90 - 0.10\alpha_h(1+2\psi)$

For members with sway buckling mode the equivalent uniform moment factor should be taken $C_{my} = 0.9$ or $C_{mz} = 0.9$ respectively

C_{my}, C_{mz} and C_{mLT} shall be obtained according to the bending moment diagram between the relevant braces points as follows:

moment factor	bending axis	points braces in direction
C_{my}	y-y	z-z
C_{mz}	z-z	y-y
C_{mLT}	y-y	y-y