Improvement on minimum distance of symbol-pair codes

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Outline

1. Section 1: Basic notations and some previous results
2. Section 2: Lower bounds on the minimum pair distance of q-ary linear cyclic codes and constacyclic codes
3. Section 3: Some specific MDS symbol-pair codes constructed by constacyclic codes
4. Section 4: Open Problems

Section 1: Basic notations and some previous results

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$$\pi(\boldsymbol{x}) = [(x_0, x_1), (x_1, x_2), \cdots, (x_{n-2}, x_{n-1}), (x_{n-1}, x_0)].$$

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- pair-distance between $\boldsymbol{x}$ and $\boldsymbol{y}$,

$$d_p(\boldsymbol{x}, \boldsymbol{y}) = d_H(\pi(\boldsymbol{x}), \pi(\boldsymbol{y})).$$

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- For any vector $\boldsymbol{x} \in \Sigma^n$, the pair weight of $\boldsymbol{x}$,

$$\omega_p(\boldsymbol{x}) = \omega_H(\pi(\boldsymbol{x})).$$

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- The minimum pair-distance of $\mathcal{C}$,

$$d_p(\mathcal{C}) = \min\{d_p(\boldsymbol{x}, \boldsymbol{y}) | \boldsymbol{x}, \boldsymbol{y} \in \mathcal{C}, \boldsymbol{x} \neq \boldsymbol{y}\}.$$

  A code of length n over Σ is called an (n, M, d_p)-symbol-pair code if its size is M and minimum pair distance is d_p.

symbol pair codes

Proposition 1. (*Singleton Bound*)(Chee *et al.*)

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An $(n, M, d_H)_q$-symbol-pair code attains the Singleton-type bound, i.e., $M = q^{n-d_H+2}$, is said to be an maximum distance separable (MDS) symbol-pair code.

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- For a nonzero element η in $\mathbb{F}_q$, the η-constacyclic shift τ_η on $\mathbb{F}_q^n$ is the shift

$$\tau_\eta(c_0, c_1, \ldots, c_{n-1}) = (\eta c_{n-1}, c_0, \ldots, c_{n-2}).$$

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  If $\eta = 1$, then $\mathcal{C}$ is just the usual *cyclic code*.

Cyclic codes and constacyclic codes

- For $\boldsymbol{x} = (x_0, x_1, \cdots, x_{n-1})$, we define

$$\boldsymbol{x}^{'} = (x_0 + x_1, x_1 + x_2, \ldots, x_{n-1} + x_0).$$

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Lemma 2. (E. Yaakobi *et al.*)

Let $\mathcal{C}$ be a linear cyclic code of dimension greater than one. Then,

$$d_p(\mathcal{C}) \geq d_H(\mathcal{C}) + \lceil \frac{d_H(\mathcal{C})}{2} \rceil.$$

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Below are our main results.

Theorem 1.

Let $\mathcal{C}$ be a q-ary linear cyclic code of dimension greater than one. Then,

$$d_p(\mathcal{C}) \geq d_H(\mathcal{C}) + \lceil \frac{d_H(\mathcal{C})}{2(q-1)} \rceil.$$

Section 2: Lower bounds on the minimum pair distance of q-ary linear cyclic codes and constacyclic codes

Theorem 2.

Let $\mathcal{C}$ be a q-ary η-constacyclic code of dimension greater than one. Then,

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Proof of Theorem 1.

For any $\boldsymbol{x} \in \Sigma^n$, let $\boldsymbol{x} = (x_0, x_1, \cdots, x_{n-1})$ be a codeword in $\mathcal{C}$. Assume that $\boldsymbol{x} \neq \boldsymbol{\alpha}, \alpha \in \mathbb{F}_q^*$. Thus for any $\lambda \in \mathbb{F}_q^*$,

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Now let

$$S_\alpha = \{i | (x_i, x_{i+1}) \neq (0,0), x_i = \alpha\}, \alpha \in \mathbb{F}_q.$$

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For any $\alpha, \beta \in \mathbb{F}_q, \alpha \neq \beta$, one has $S_\alpha \cap S_\beta = \emptyset$ and $w_p(\boldsymbol{x}) = \sum_{\alpha \in \mathbb{F}_q} |S_\alpha| = w_H(x) + |S_0|$. Thus, one has

$$\begin{aligned} \sum_{\lambda \in \mathbb{F}_q^*} w_H(\boldsymbol{x}_\lambda') &= 2(q-1)|S_0| + (q-2) \sum_{\alpha \in \mathbb{F}_q^*} |S_\alpha| \\ &= 2(q-1)|S_0| + (q-2) w_H(\boldsymbol{x}). \end{aligned}$$

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Then we can deduce that

$$|S_0| = \frac{1}{2(q-1)} \sum_{\lambda \in \mathbb{F}_q^*} w_H(\boldsymbol{x}_\lambda^{'}) - \frac{q-2}{2(q-1)} w_H(\boldsymbol{x}).$$

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& \geq \frac{d_H}{2} - \frac{q-2}{2(q-1)} d_H + d_H \\
& = \frac{d_H}{2(q-1)} + d_H.
\end{aligned}
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Hence,

$$w_p(\boldsymbol{x}) \geq \lceil \frac{d_H}{2(q-1)} \rceil + d_H.$$

Proof of Theorem 2.

The proof of Theorem 2 implied by Theorem 1.

Section 2: Lower bounds on the minimum pair distance of q-ary linear cyclic codes and constacyclic codes

Lemma 2. (E. Yaakobi *et al.*)

Let $\mathcal{C}$ be a linear cyclic code of dimension greater than one. Then,

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Let $\boldsymbol{x} = (x_0, x_1, \cdots, x_{n-1}) \in \Sigma^n$. Our goal is to calculate $w_p(\boldsymbol{x})$, namely,

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$S_0 = \{i : (x_i, x_{i+1}) \neq (0, 0) \text{ and } x_i = 1\}$,

$S_1 = \{i : (x_i, x_{i+1}) = (0, 1)\}$.

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Hence, $|S_0| = \omega_H(\boldsymbol{x})$, $S_0 \cap S_1 = \emptyset$, and $\omega_p(\boldsymbol{x}) = |S_0| + |S_1|$.

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For all $0 \leq i \leq n-1$, $i \in S_1$, we get

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Note that for any $\boldsymbol{x} \in \Sigma^n$,

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and the sum of the cardinality of the two sets is $\omega_H(\boldsymbol{x}^{'})$. Hence, $|S_1| = \frac{\omega_H(\boldsymbol{x}^{'})}{2}$ and

$$\omega_p(\boldsymbol{x}) = |S_0| + |S_1| = \omega_H(\boldsymbol{x}) + \frac{\omega_H(\boldsymbol{x}^{'})}{2}.$$

Section 3: Some specific MDS symbol-pair codes constructed by constacyclic codes

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Example

(1) Let $\mathcal{C}$ be a $[23, 3, 19]_5$ η-constacyclic code, there exists an MDS $[23, 22]_5$-symbol-pair code. (2) Let $\mathcal{C}$ be a $[11, 5, 6]$ η-constacyclic code over $\mathbb{F}_3$, there exists an MDS $(11, 8)_3$-symbol-pair code.

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- Is there any optimal code constructions by using constacyclic codes?

Thanks for your attention!