A Linearly Homomorphic Signature Scheme From Weaker Assumptions

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Organization

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- Motivation
- Homomorphic Signature Schemes
- Our Construction
- Combining our Construction with Homomorphic Encryption
- Conclusion

Motivation

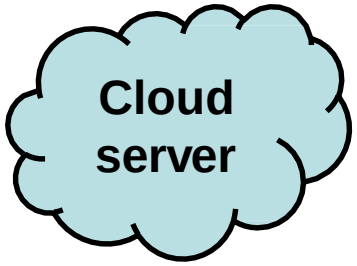
Alice

Cloud server

Motivation

Motivation

Alice



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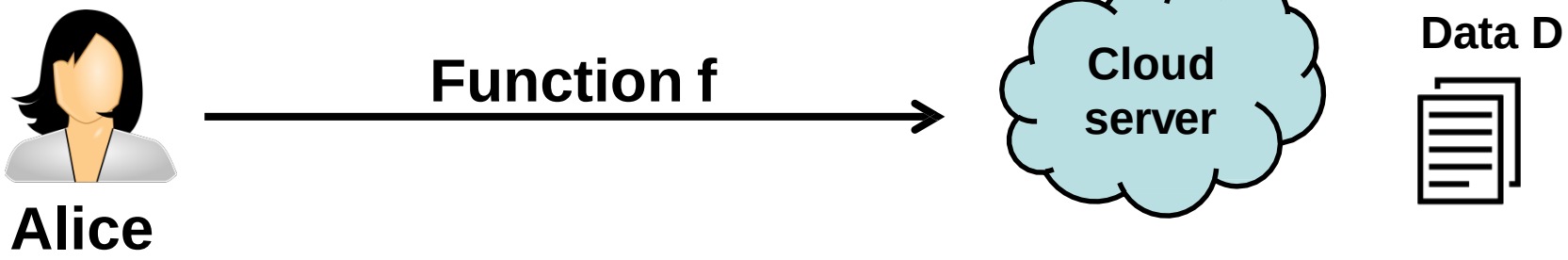
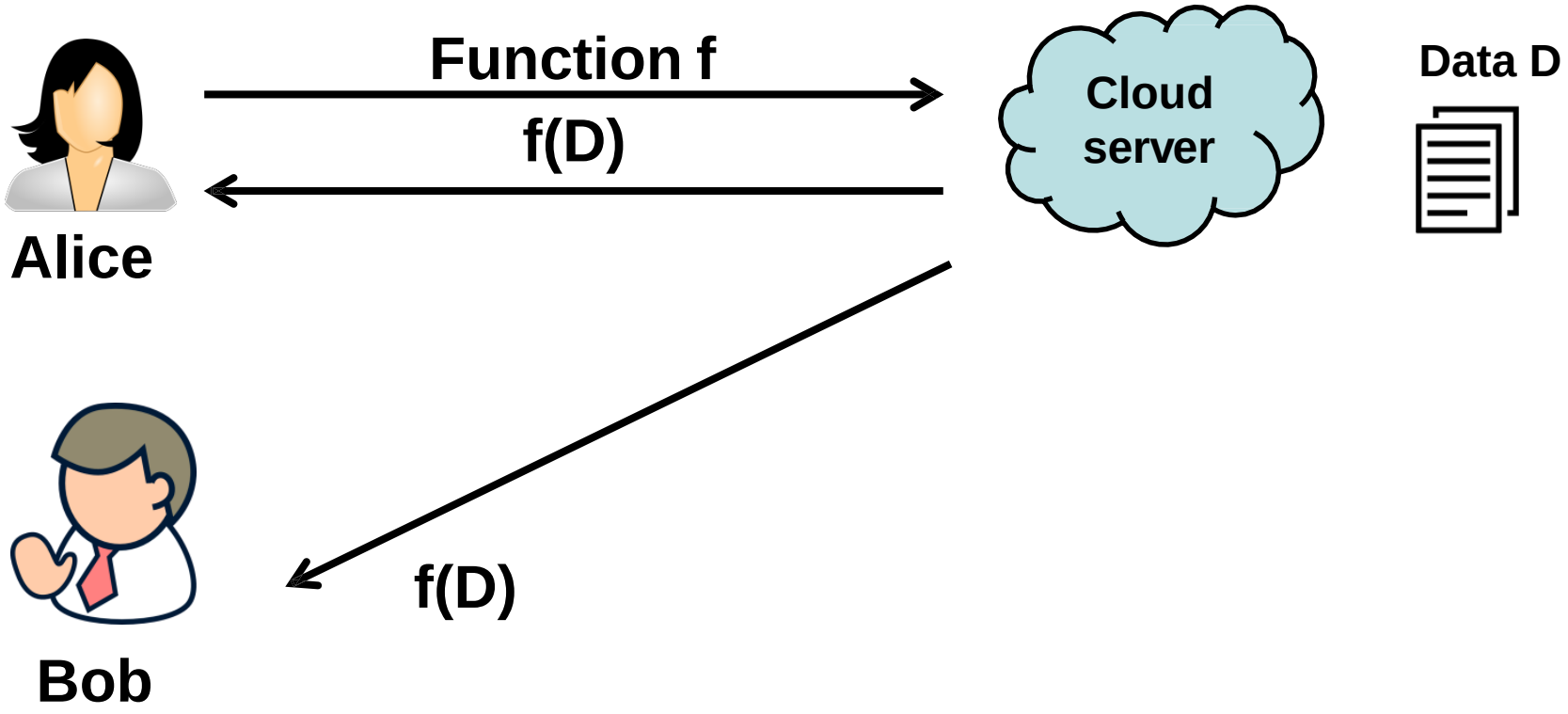
Motivation



Motivation



Motivation

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- Alice or Bob does not trust the cloud
  - Computation result has to be verifiable
  - **Solution**: homomorphic signature scheme
- Sensitive data
  - Data has to be confidential
  - **Solution**: homomorphic encryption scheme

Homomorphic Signature Schemes

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- Messages are stored in datasets identified by an identifier Δ
- Typically, the dataset size is fixed by a value k
- Functions can only be evaluated over messages in the same dataset

	Δ_1	Δ_2	Δ_3
$i = 1$:	m_1	m^*_1	m'_1
$i = 2$:	m_2	m^*_2	m'_2
$i = 3$:	m_3	m^*_3	m'_3
...	...	...	...
...	...	...	...
...	...	...	...
$i = k$:	m_k	m^*_k	m'_k

Homomorphic Signatures

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$\mathsf{HKeyGen}(1^\lambda)$: Output: key pair $(\mathsf{sk}, \mathsf{pk})$.

$\mathsf{HSign}(\mathsf{sk}, \Delta, \tau, m)$: Output: Signature σ.

$\mathsf{HEval}(\mathsf{pk}, f, \boldsymbol{\sigma})$: Output: Signature σ.

$\mathsf{HVerify}(\mathsf{pk}, (f, \tau_1, \ldots, \tau_n, \Delta), m, \sigma)$: Output: 0 or 1.

Homomorphic Signatures

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Security Intuition: No signatures for $m \neq f(m_1, \ldots, m_k)$ can be derived.

Weak security: Adversary can not query messages adaptively.

Strong security: Adversary **can** query messages adaptively.

Homomorphic Signatures

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Existing solutions for strongly secure linearly homomorphic signatures all use „strong“ assumptions.

[BFKW09]: 2-out-of-3 CDH Assumption
[CFW11]: Strong RSA Assumption
[ALP13]: k-simultaneous Flexible Pairing Assumption
[CFN15]: Flexible DH Inversion
…
This work: CDH Assumption

„Weak“ vs „Strong“Assumptions

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Unique solution, example CDH:

Given: $(\mathbb{G}, g, g^a, g^b)$

Compute: g^{ab}

Class of solutions, example 2-out-of 3-CDH:

Given: $(\mathbb{G}, g, h, g^a)$

Compute: (g^r, h^{ar}) for some r

Preliminaries

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Bilinear Groups: $e : \mathbb{G}_1 \times \mathbb{G}_2 \rightarrow \mathbb{G}_T$

$$e(g_1, g_2) \neq 1_{\mathbb{G}_T} \forall g_1 \neq 1_{\mathbb{G}_1}, g_2 \neq 1_{\mathbb{G}_2}$$
$$e(g_1^a, g_2^b) = e(g_1, g_2)^{ab}$$

If $\mathbb{G}_1 \simeq \mathbb{G}_2$, we write: $\mathbb{G}$

We will describe our scheme in symmetric groups (for notations sake).
It works in (more efficient) asymmetric groups as well.

Messages: $m = \begin{pmatrix} m[1] \\ m[2] \\ \dots \\ m[T] \end{pmatrix} \in \mathbb{Z}_q^T$

Our Solution

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$\mathsf{HKeyGen}(1^\lambda)$: Obtains a bilinear group $\mathsf{bgp} = (q, \mathbb{G}, \mathbb{G}_T, g, e)$, and samples $k + T$ elements $R_1, \ldots, R_k, h_1, \ldots, h_T \leftarrow \mathbb{G}$. Additionally it generates a key pair $(\mathsf{sk}', \mathsf{pk}') \leftarrow \mathsf{KeyGen}'(1^\lambda)$ of a regular signature scheme and a key $K \leftarrow \mathcal{K}$ for a pseudorandom function $F : \mathcal{K} \times \{0,1\}^* \rightarrow \mathbb{Z}_q$. Output $\mathsf{sk} = (\mathsf{sk}', K)$ and $\mathsf{pk} = (\mathsf{pk}', \mathsf{bgp}, \{h_j\}_{j=1}^T, \{R_i\}_{i=1}^k)$.

$\mathsf{HSign}(\mathsf{sk}, \Delta, i, m)$: It runs $z \leftarrow F_K(\Delta)$ and computing $Z = g^z$. It binds Z to the dataset identifier Δ by using the regular signature scheme, i.e. it sets $\sigma_\Delta \leftarrow \mathsf{Sign}'(\mathsf{sk}', (Z|\Delta))$. Then, it computes $\Lambda \leftarrow (R_i \cdot \prod_{j=1}^T h_j^{m[j]})^z$ and returns the signature $\sigma = (\sigma_\Delta, Z, \Lambda)$.

Our Solution

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$\mathsf{HEval}(\mathsf{pk}, f, \boldsymbol{\sigma})$: It computes $\Lambda = \prod_{i=1}^{k} \Lambda_i^{f_i}$, and returns the signature $\sigma = (Z_1, \sigma_{\Delta,1}, \Lambda)$.

$\mathsf{HVerify}(\mathsf{pk}, (f, \tau_1, \ldots, \tau_k, \Delta), m, \sigma)$: It checks if $\mathsf{Verify}'(\mathsf{pk}', (Z|\Delta), \sigma_\Delta) = 1$ and $e\left(R \cdot \prod_{j=1}^{T} h_j^{m[j]}, Z\right) = e(\Lambda, g)$, where $R \leftarrow \prod_{i=1}^{k} R_i^{f_i}$. If both checks pass, it outputs 1, otherwise, it returns 0.

Our Solution

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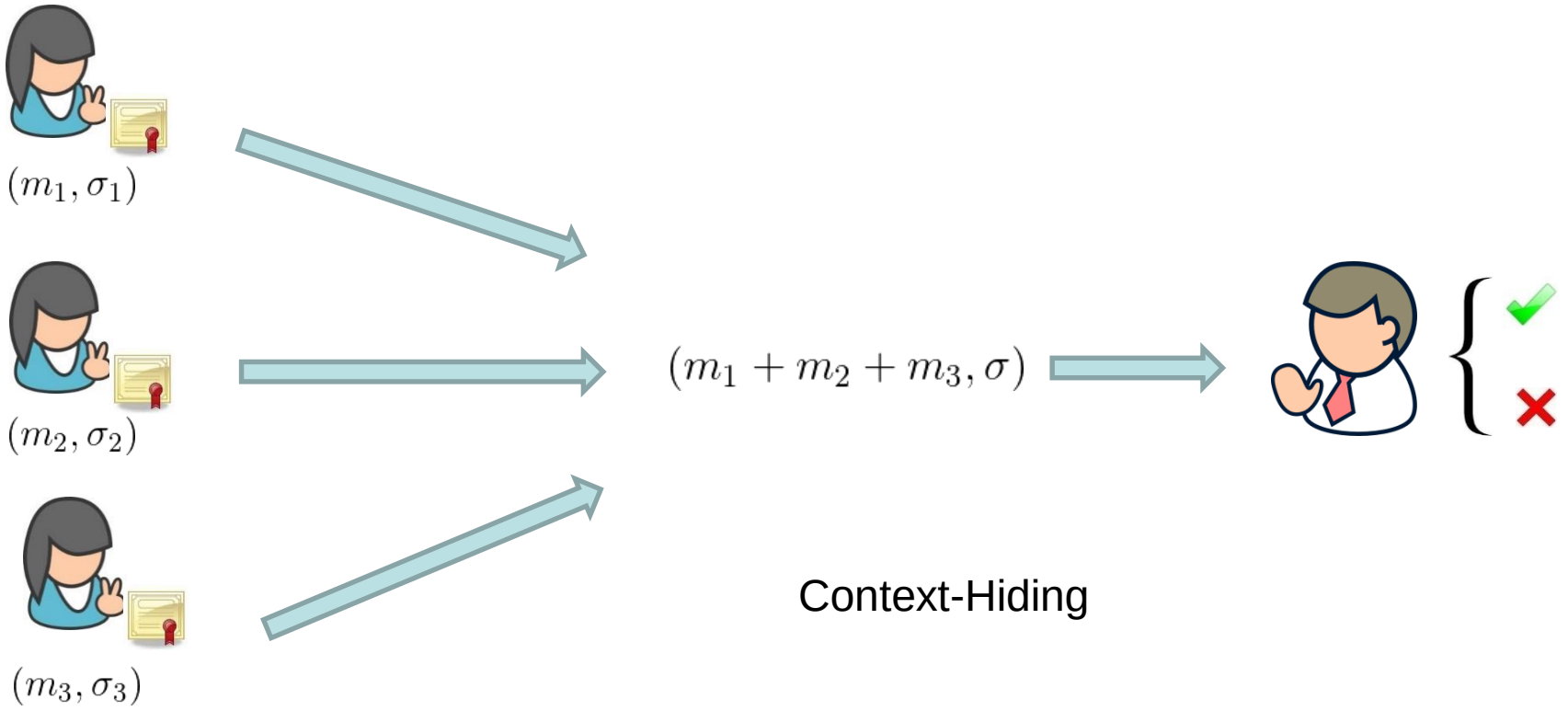
Strongly secure under CDH in $\mathbb{G}$ in the standard model, if *F* is pseudorandom and *Sig* is an unforgeable (conventional) signature scheme.

Constant-time verification, after a one time preprocessing.

Perfectly context hiding.

Can be combined with homomorphic encryption.

Input Privacy



Linearly Homomorphic Authenticated Encryption with Public Verifiability (LAEPuV)

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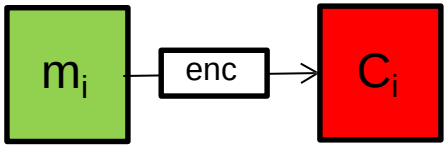

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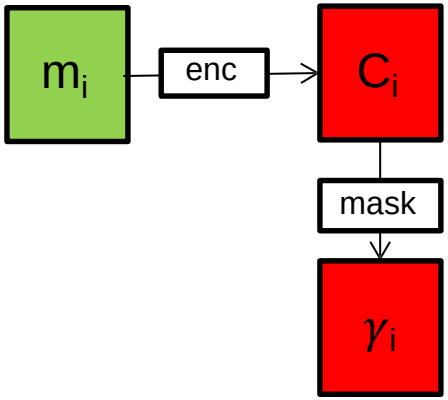

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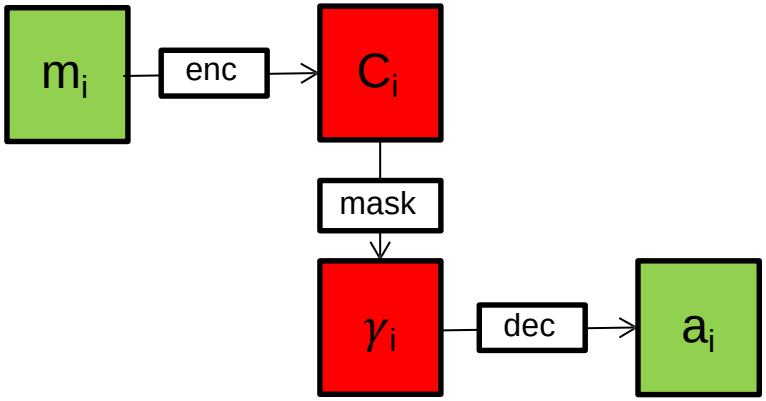

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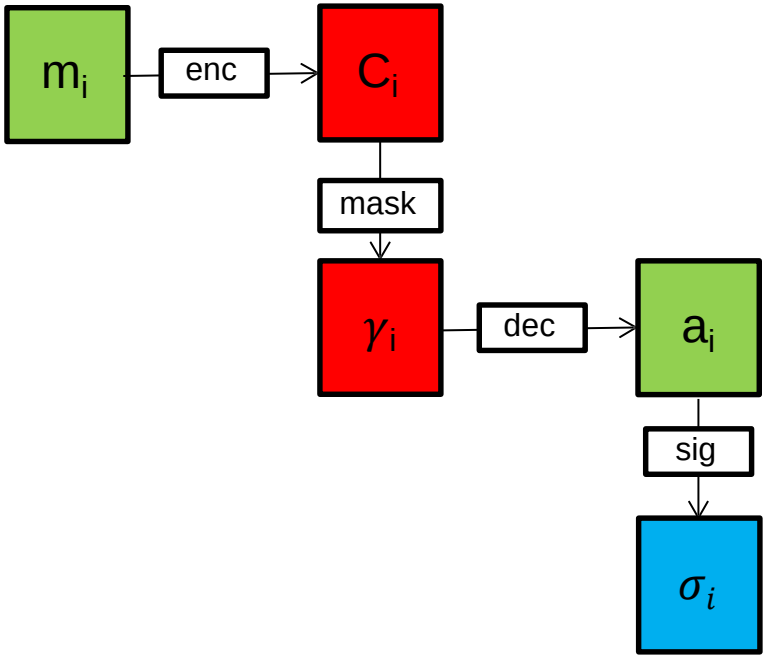

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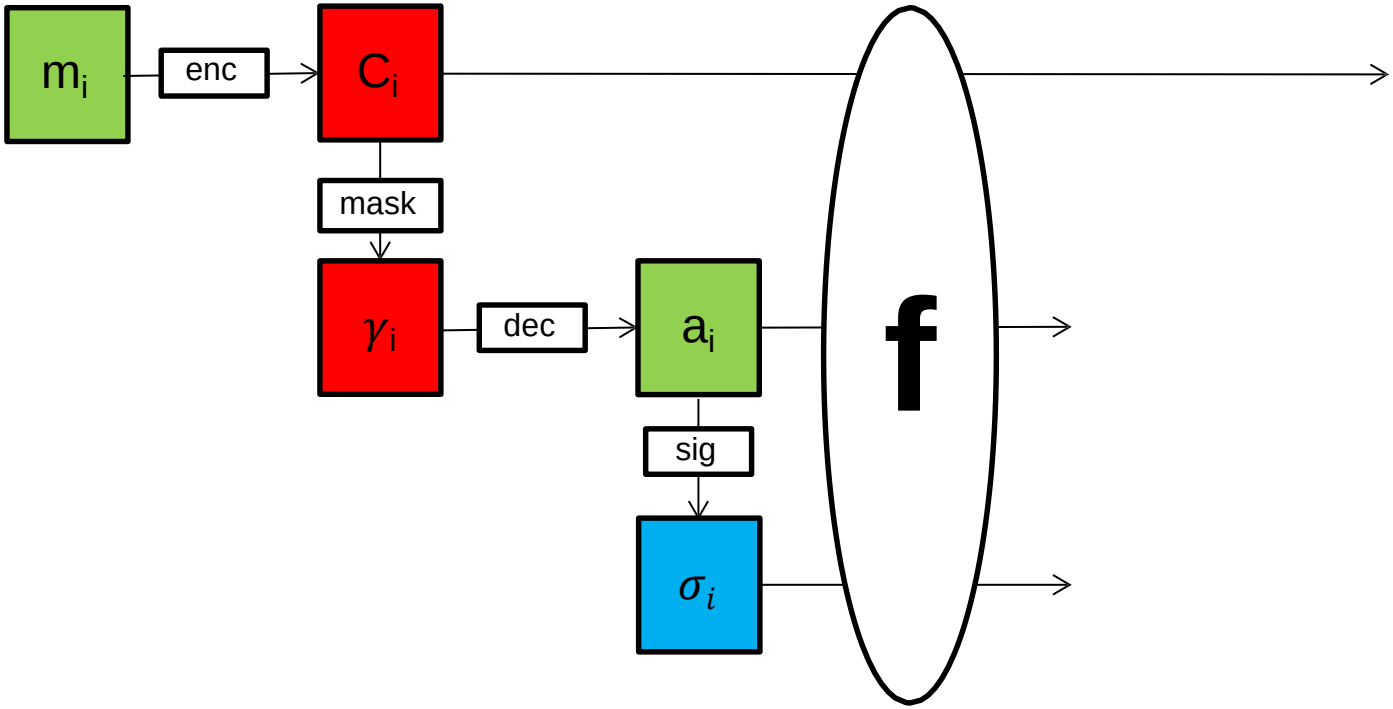

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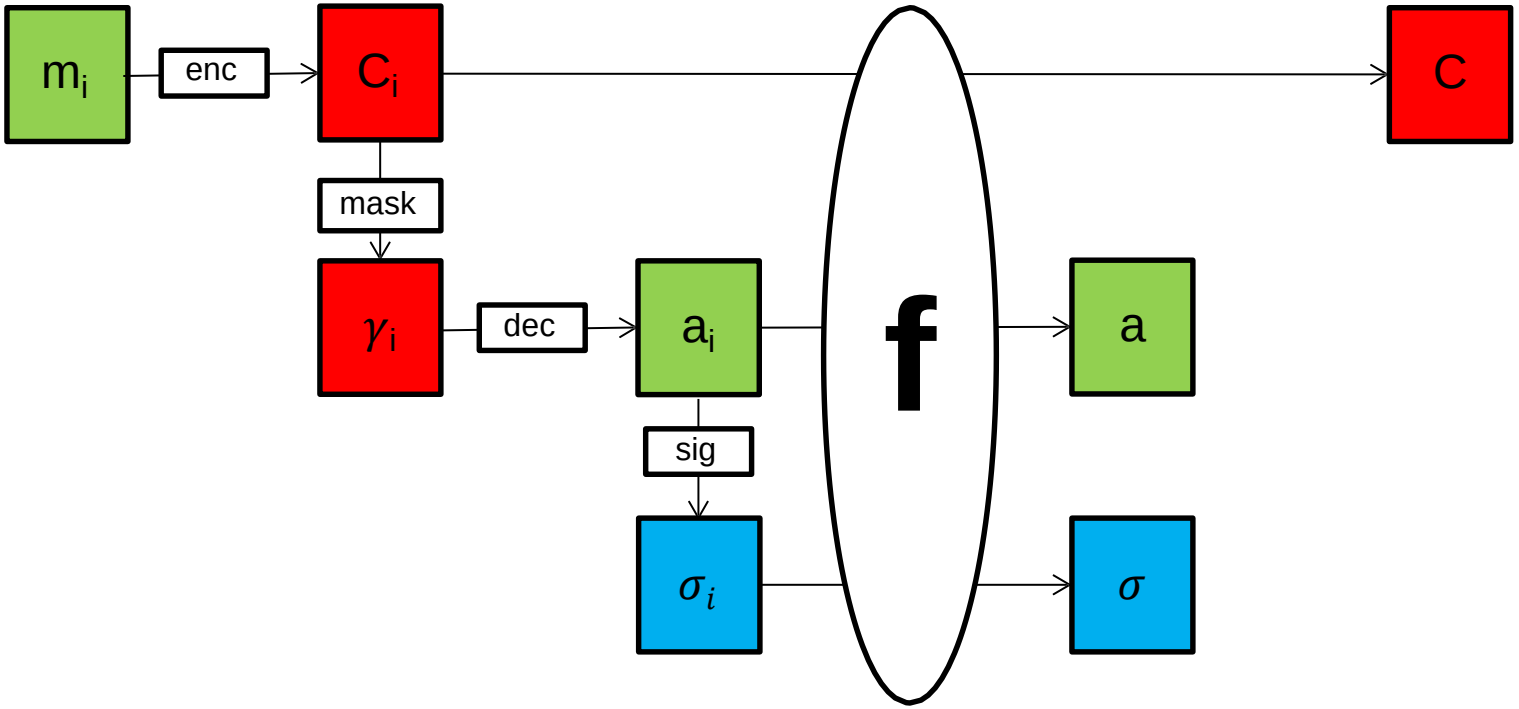

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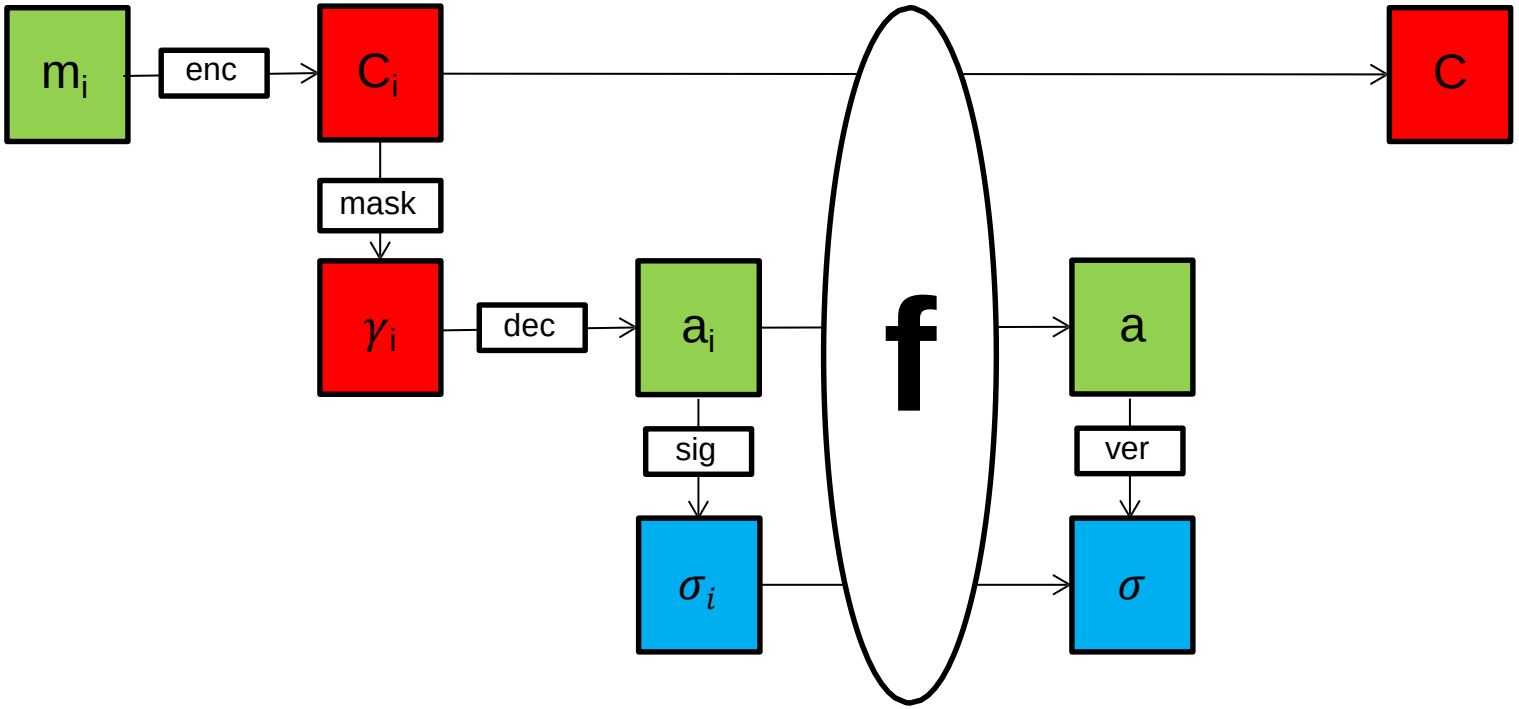

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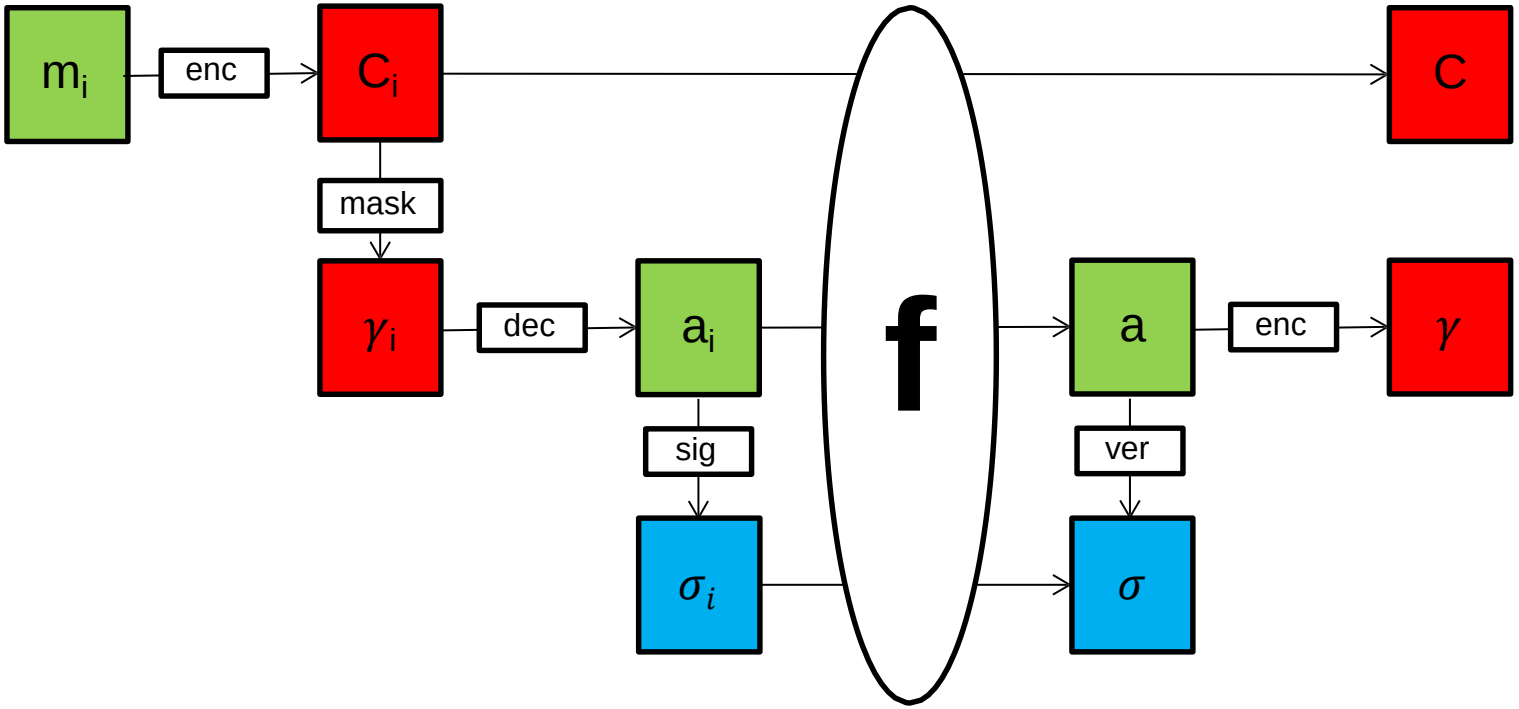

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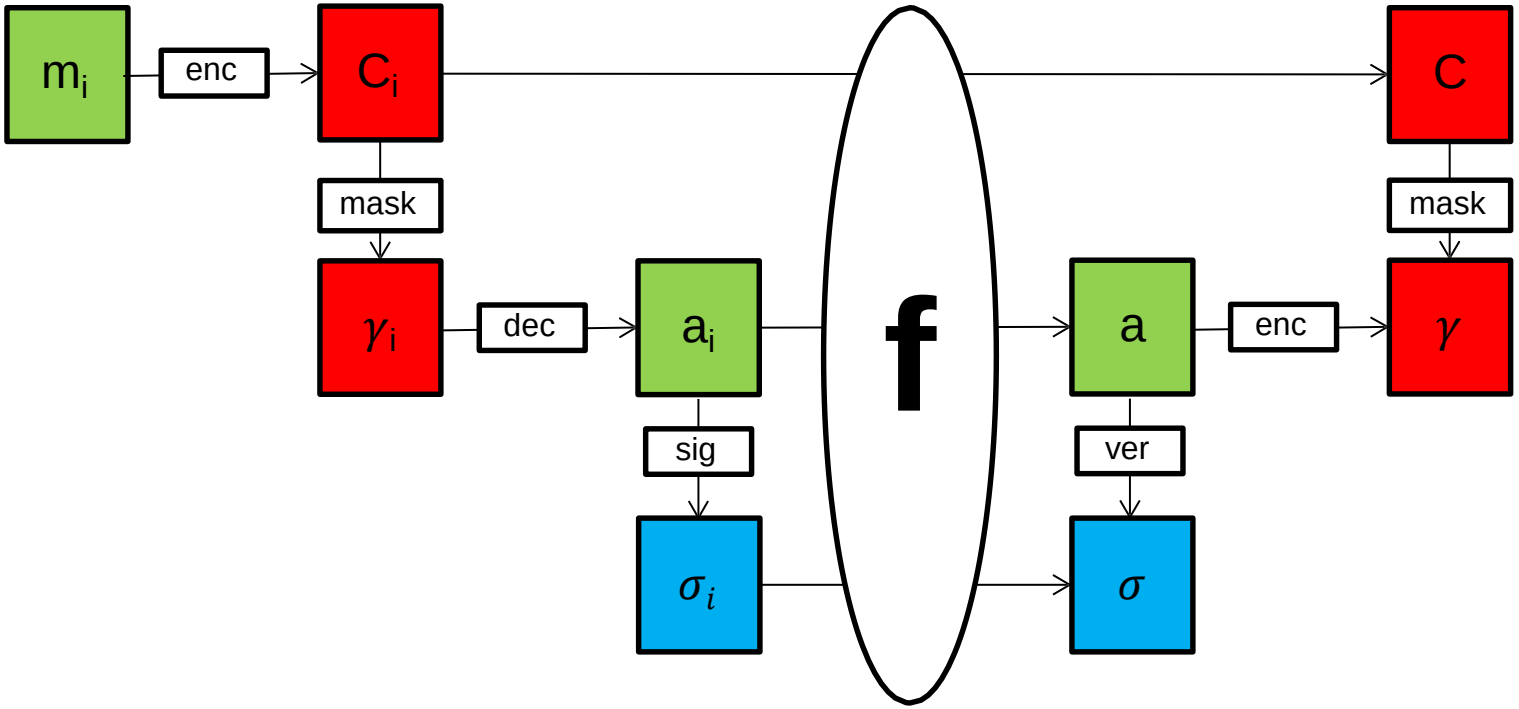

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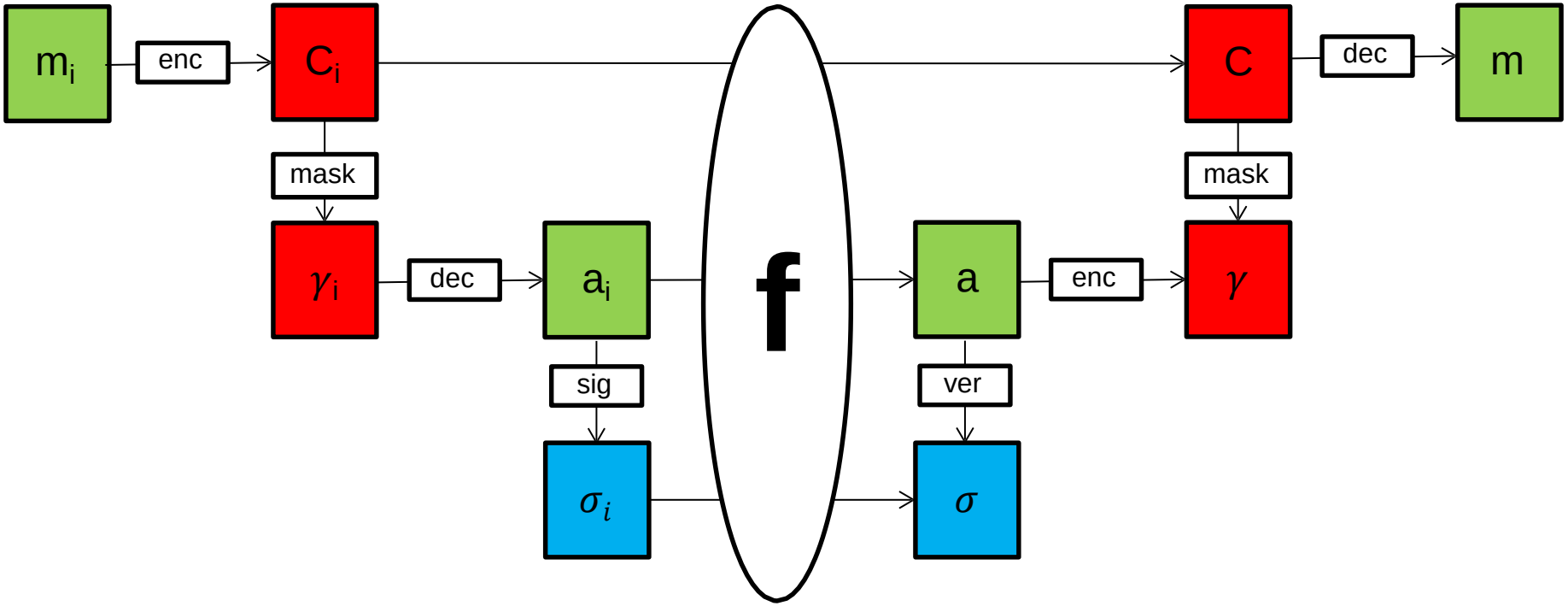


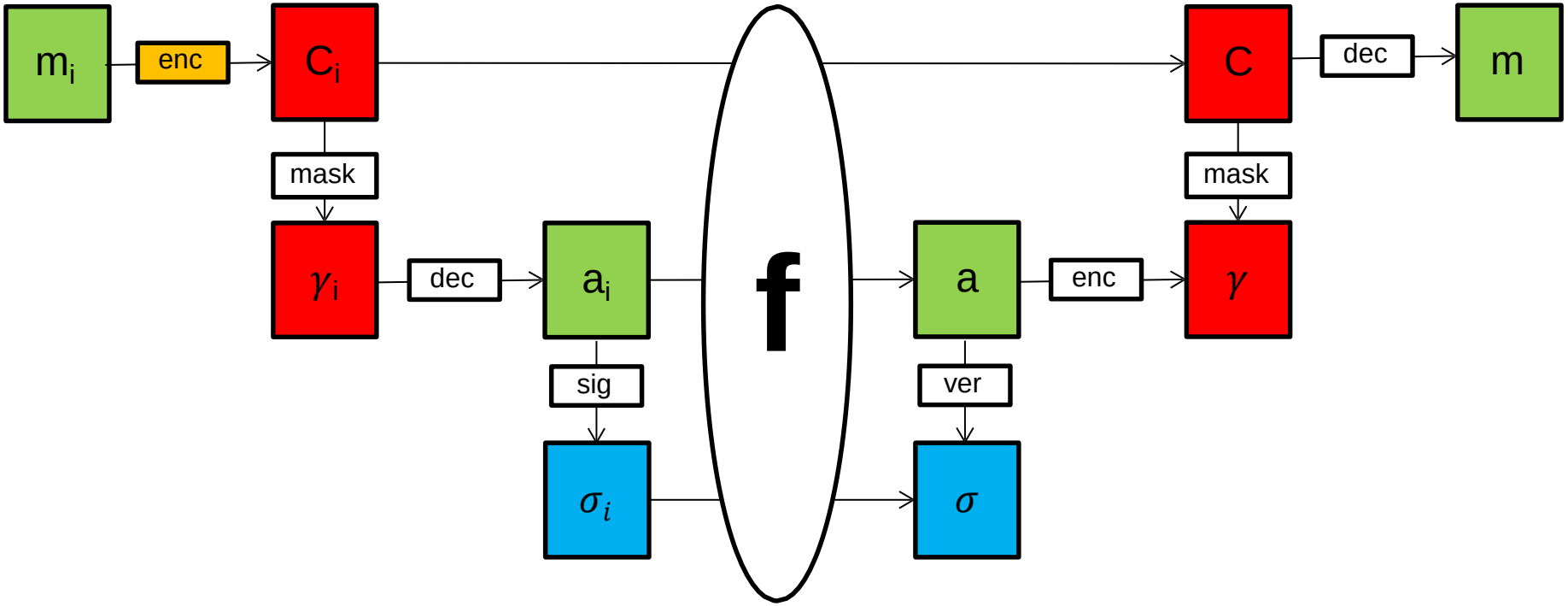
Paillier Encryption

$$C = g^m \cdot r^n \bmod n^2 \text{ with } n = pq$$

Linearly Homomorphic

Public Key Encryption

LAEPuV



$$C_i = Enc(m_i)$$

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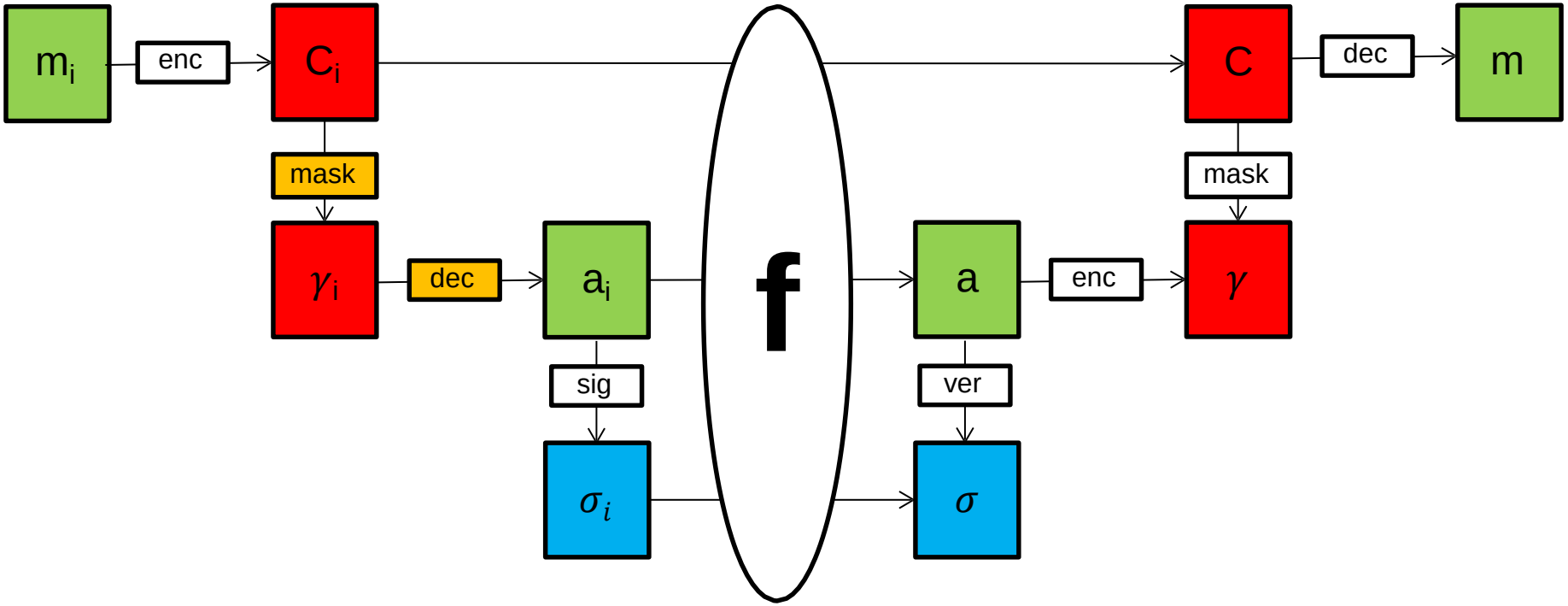

$$S \leftarrow H(\Delta|i) \qquad a_i \leftarrow Dec(C_i S_i)$$

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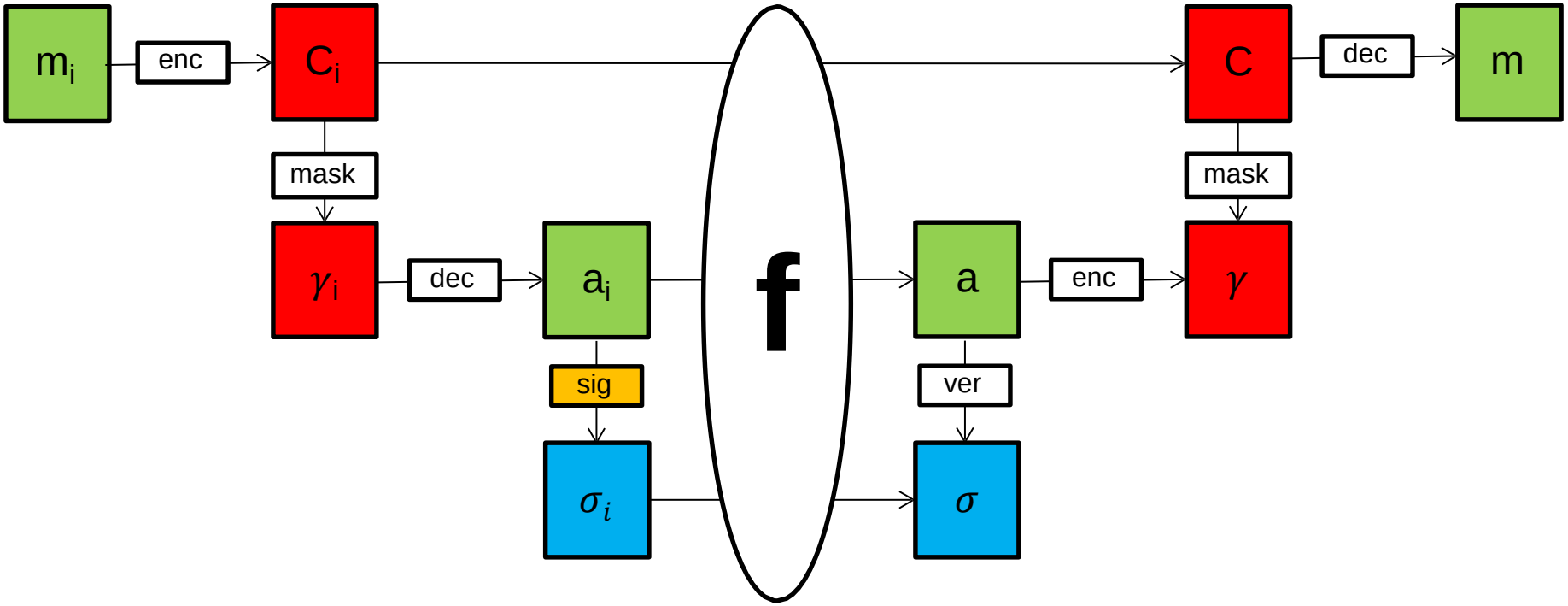
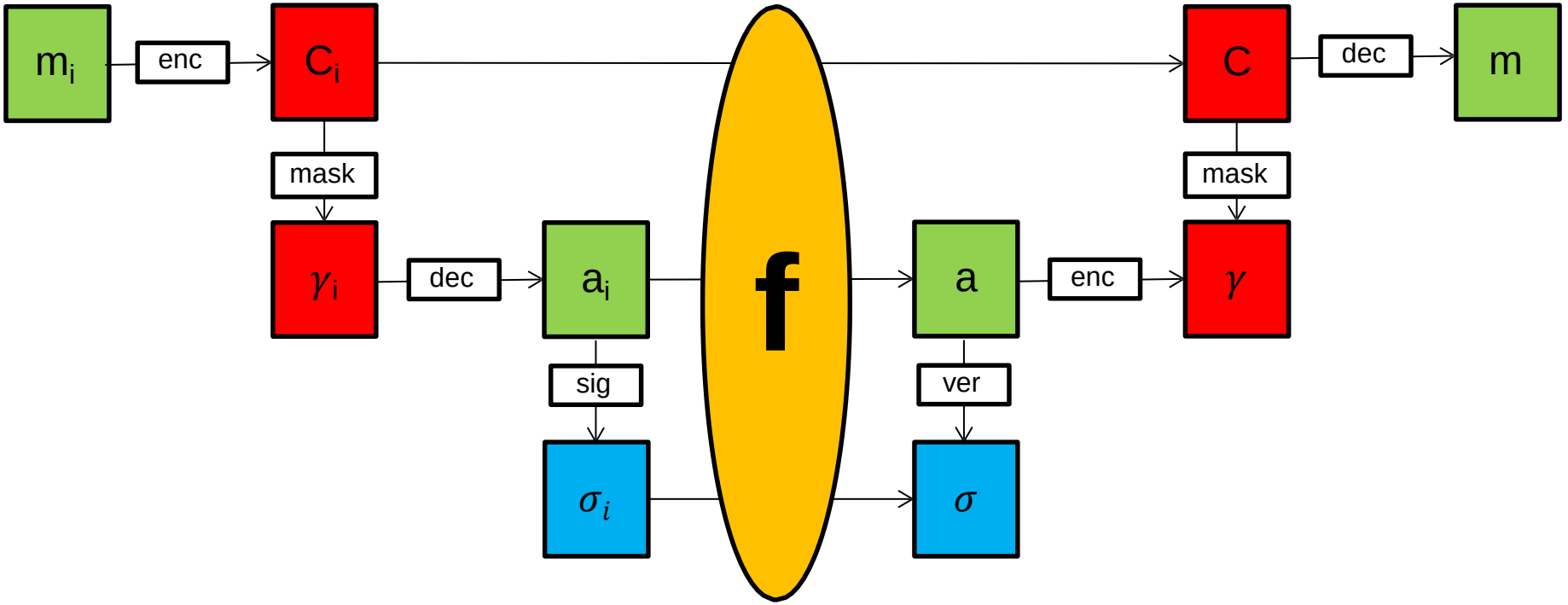


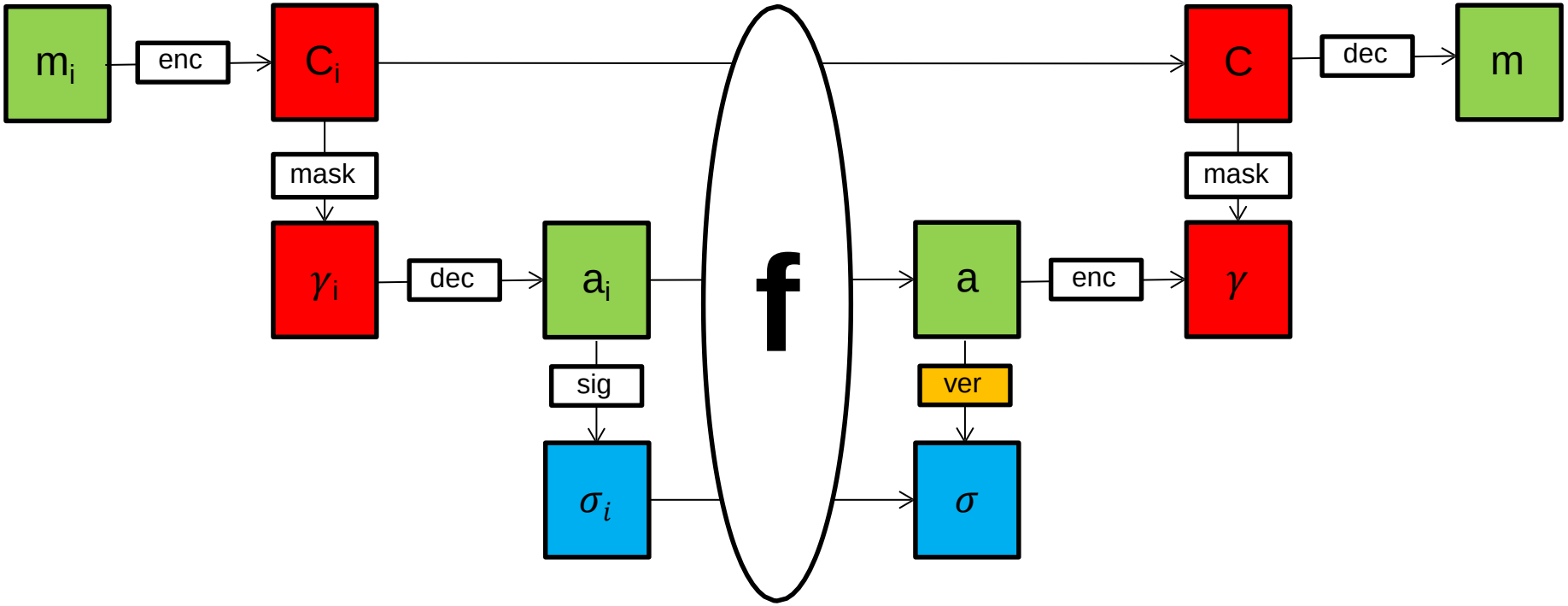
$$\Lambda \leftarrow (R_i \cdot h^{a_i})^z$$

LAEPuV



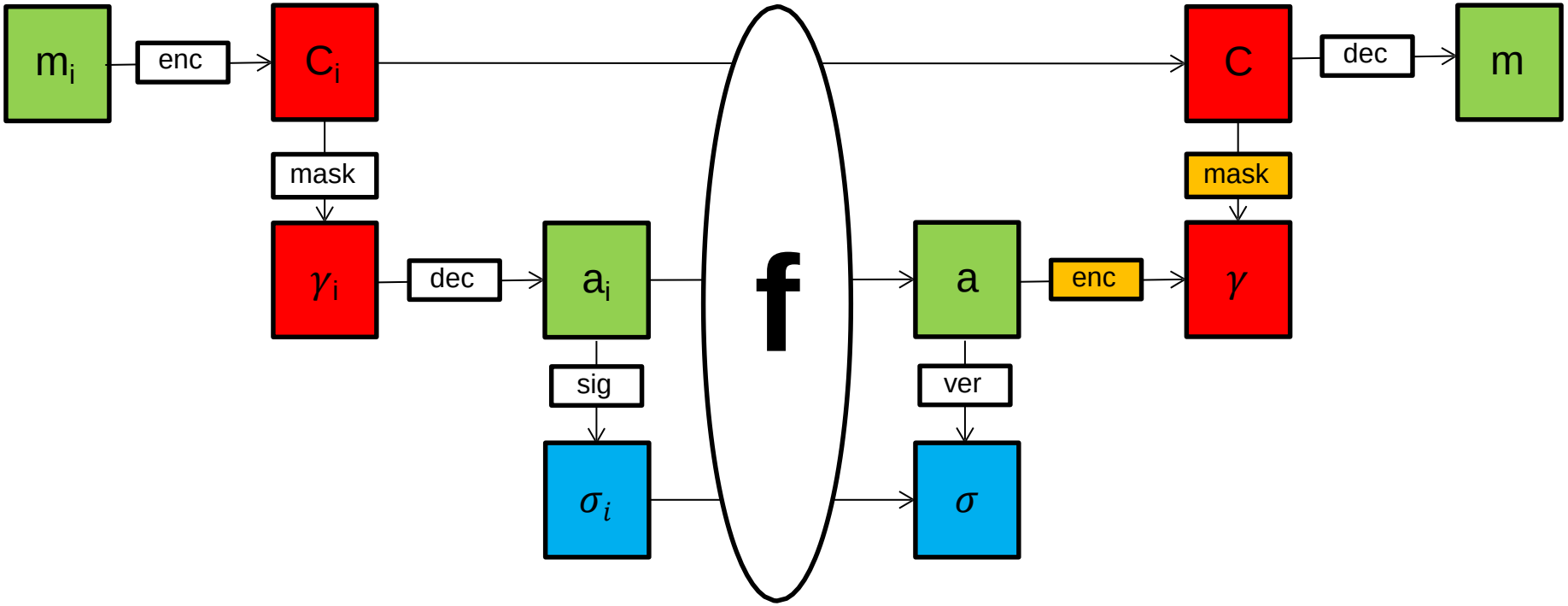
$$m \leftarrow \sum_{i=1}^{n} f_i m_i \qquad a \leftarrow \sum_{i=1}^{n} f_i a_i \qquad \Lambda \leftarrow \prod_{i=1}^{n} \Lambda_i^{f_i} \qquad C \leftarrow \prod_{i=1}^{n} C_i^{f_i}$$

LAEPuV



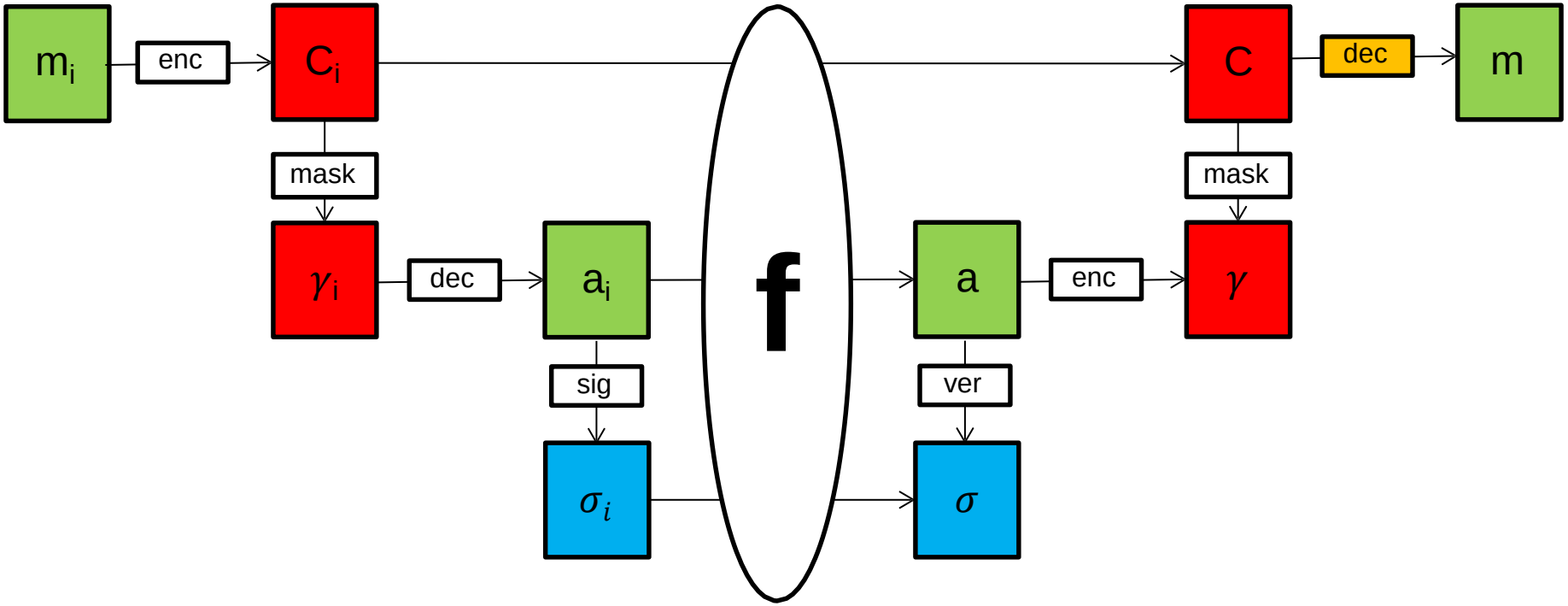
$$e(Rh^a, Z) = e(\Lambda, g)$$

LAEPuV



$$S \leftarrow \prod_{i=1}^{n} H(\Delta|i)^{f_i} \qquad Enc(a) = SC$$

LAEPuV



$$m = Dec(C)$$

Conclusion

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Strongly secure under CDH in $\mathbb{G}$ in the standard model, if *F* is pseudorandom and *Sig* is an unforgeable (conventional) signature scheme.

Constant-time verification, after a one time preprocessing.

Information-theoretically context hiding (input privacy).

Can be extended to a LAEPuV scheme (output privacy).

Results

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Thank you for your attention!

Questions?