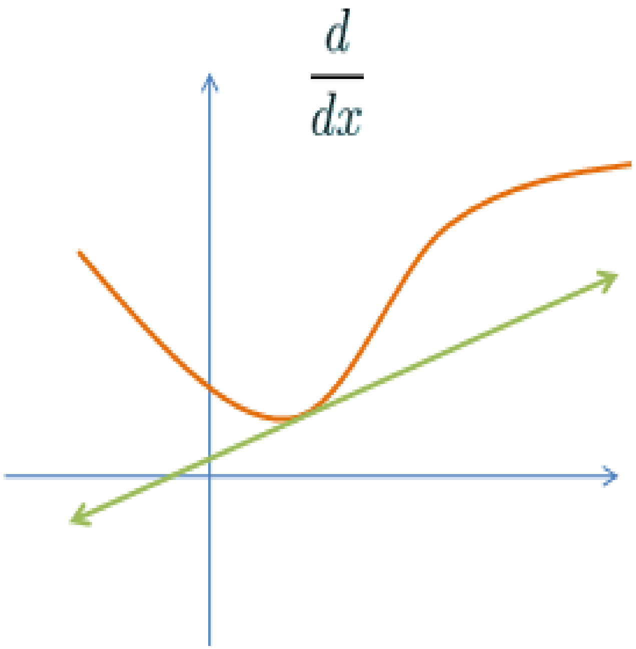
Differentiation



Who is this workshop for?

- Scientists-physicists, chemists, social scientists etc.
- Engineers
- Economists
- Calculus is used almost everywhere!

What we will cover:

- What is differentiation? Differentiation from first principles.
- Differentiating simple functions
- Product and quotient rules
- Chain rule
- Using differentiation tables
- Finding the maxima and minima of functions

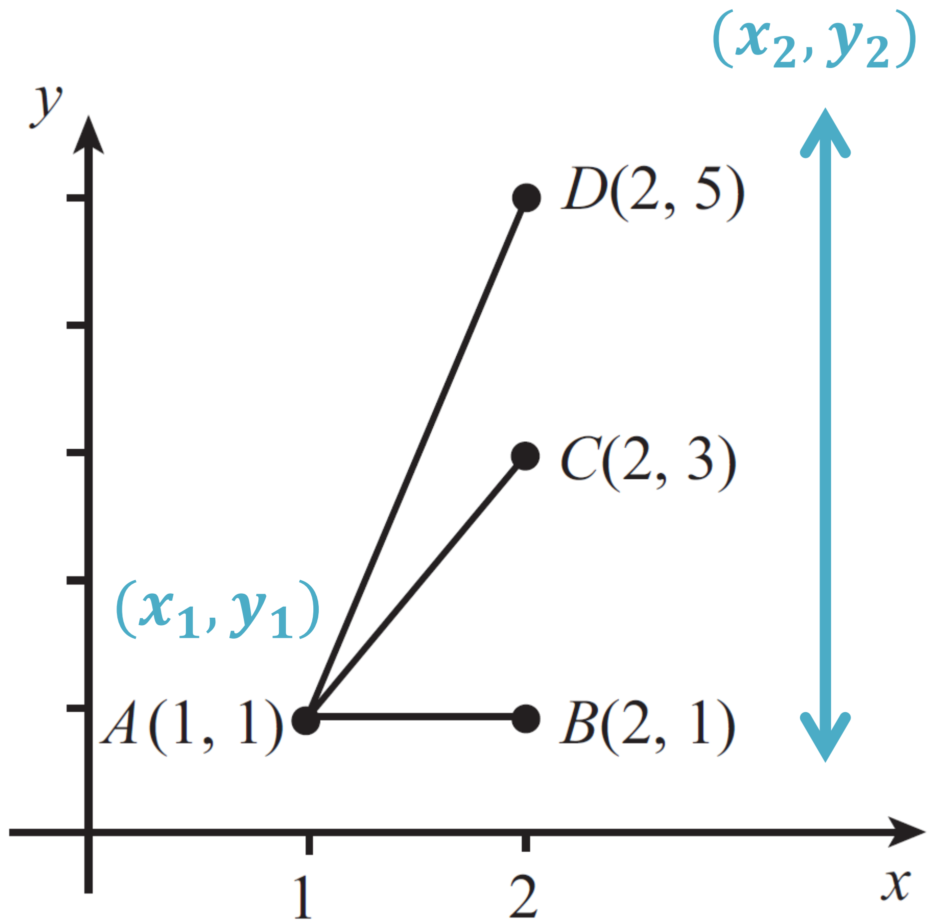
We could also cover:

- Second order differentiation
- Partial differentiation
- Whatever topic you want to know more about…

Slope of a straight line

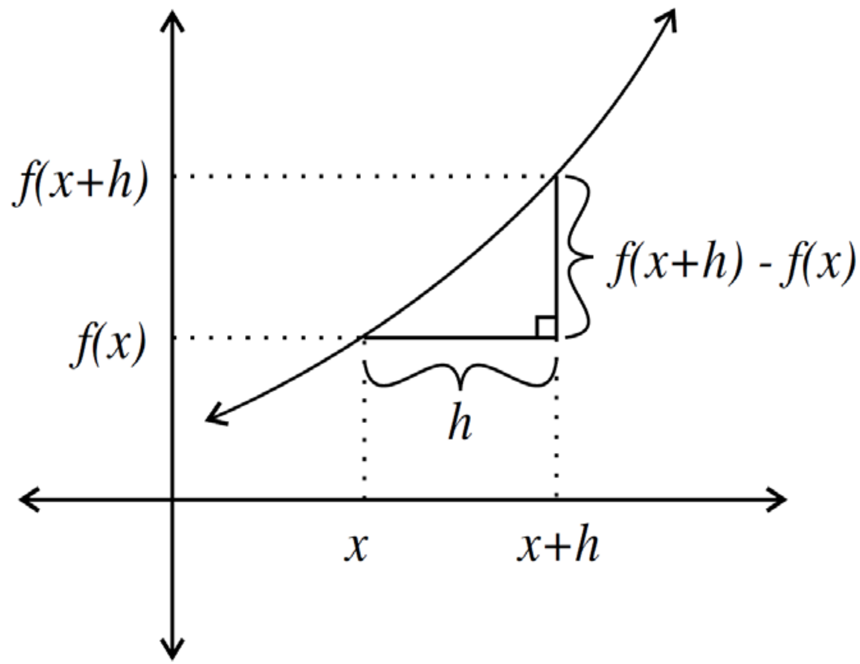
- The gradient is the 'rise over the run'

$$m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$$



What is differentiation?

- In general have function $f(x)$
- In this case:

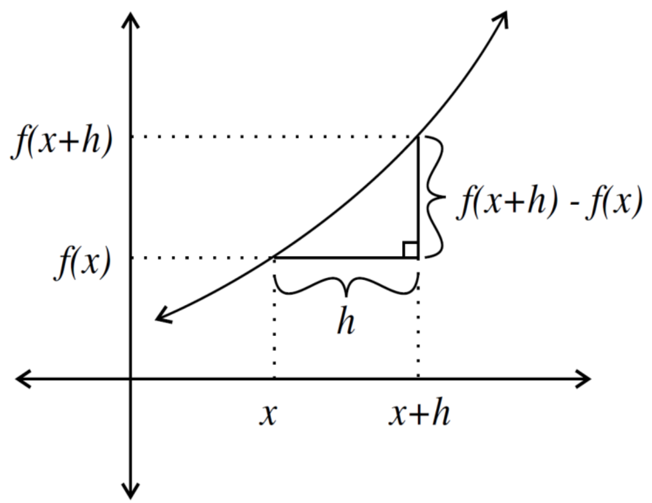


$(x_1, y_1) = (x, f(x))$
$(x_2, y_2) = (x + h, f(x + h))$

$$m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{f(x+h) - f(x)}{(x+h) - x}$$

What is differentiation?

- What happens as h shrinks?
- m gets better at approximating the gradient at $f(x)$. We call the limit when $h \to 0$ the differential:



$$\frac{dy}{dx} = f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

What is differentiation?
Examples

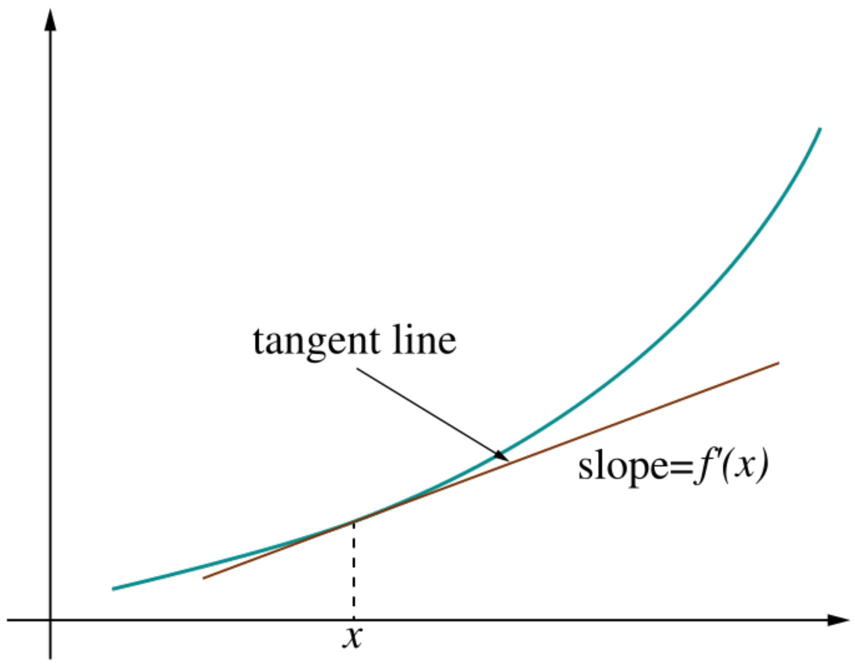
(i) What is the **slope**, *m*, of $f(x) = 2x + 5$ at the points $(x_1, y_1) = \big(1, f(1)\big)$ and $(x_2, y_2) = \big(1.01, f(1.01)\big)$?

(ii) Let $f(x) = x^2$. Find the **slope** joining $\big(x, f(x)\big)$ and and $\big(x + h, f(x + h)\big)$ if $h = 0.1$ and $x = 1$.

(iii) Using the **formal definition of the derivative** (previous slide) find the derivative of $f(x) = x^2 + 3$.

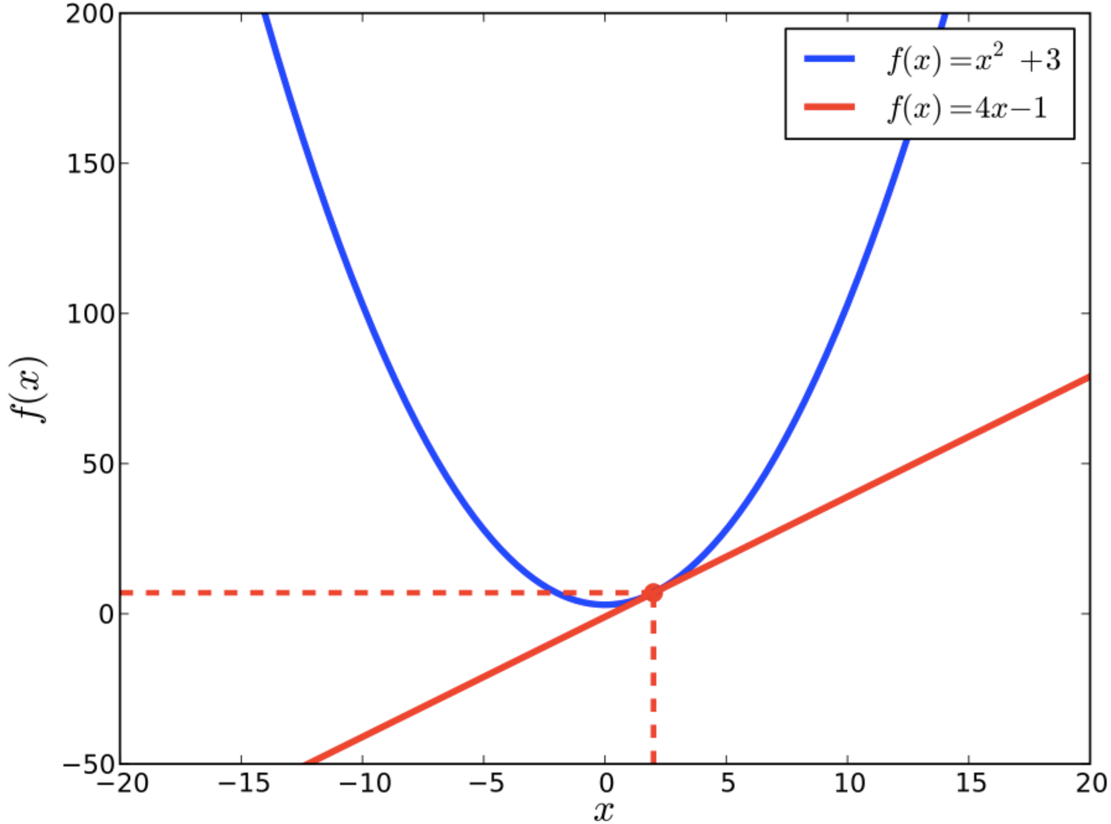
The Tangent

The **derivative** at a position x is the gradient of the **tangent to the curve.**



The Tangent
Example

Find the gradient of the tangent at the point (2,7) in the following example:



Basic formulae for differentiation

Constant:

$$y = C \quad \rightarrow \quad \frac{dy}{dx} = 0$$

Polynomial:

$$y = Cx^n \quad \rightarrow \quad \frac{dy}{dx} = nCx^{n-1}$$

Sum:

$$y = f + g \quad \rightarrow \quad \frac{dy}{dx} = \frac{df}{dx} + \frac{dg}{dx}$$

Worked Examples

(i) Differentiate $y = 5$

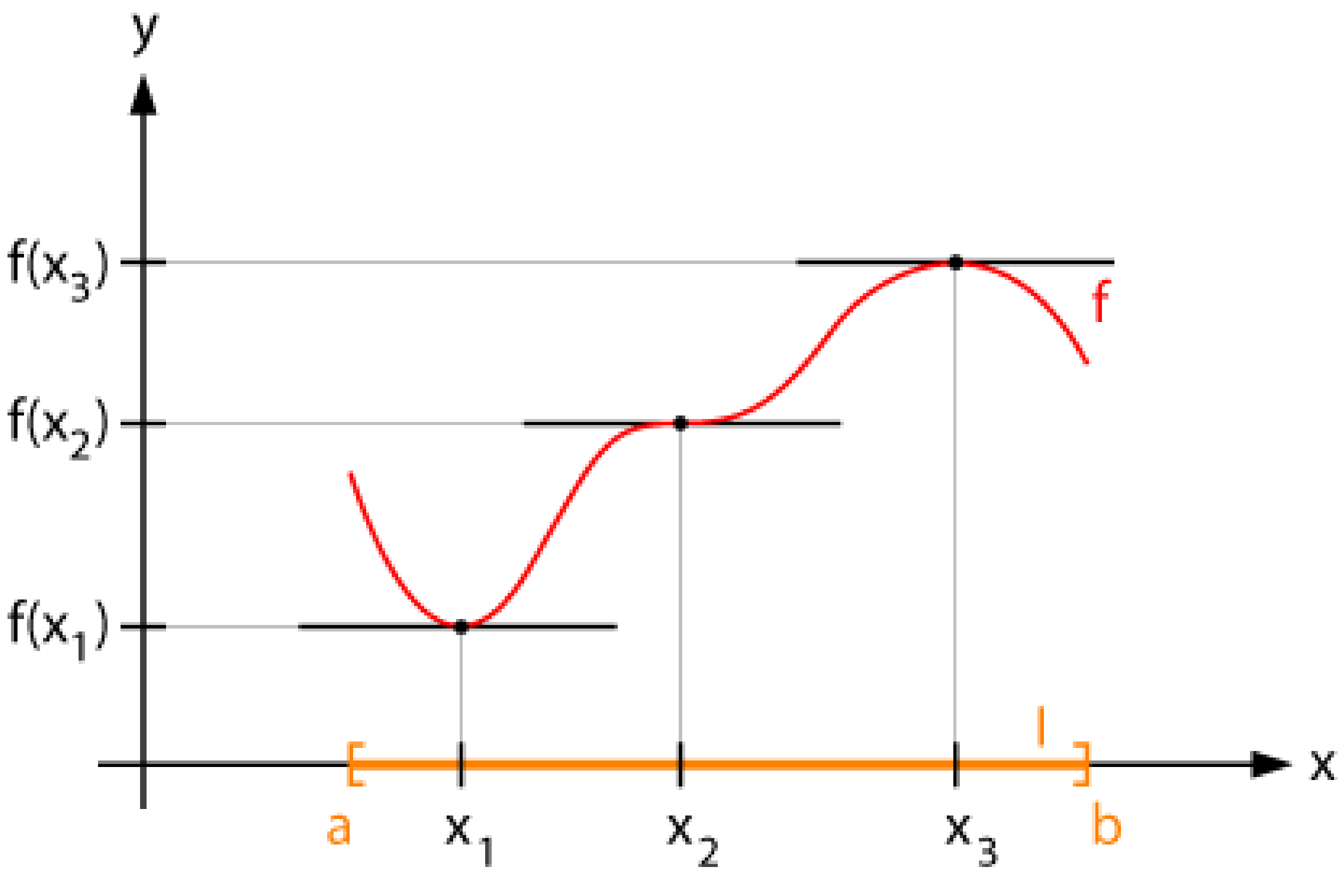
(ii) When $x = 5$ what is the rate of change of y w.r.t. x for the curve $y = -x^3 + \frac{1}{x} - 90$?

(iii) If $y = 3x^4 - x^2 + 43x - 2$ find dy/dx

Stop. Try some examples:

C1

Finding the maxima and minima



Finding the maxima and minima

- Differentiate
- Set dy/dx=0
- Solve for x
- Use original equation to find y

Find the stationary point of $y = 3x^2 - 12x + 2$

Stop. Try some examples:

C6

Table of Derivatives

$y = f(x)$	$\frac{\mathrm{d}y}{\mathrm{d}x} = f'(x)$
k, any constant	0
x	1
x^2	$2x$
x^3	$3x^2$
x^n, any constant n	nx^{n-1}
e^x	e^x
e^{kx}	$k\mathrm{e}^{kx}$
$\ln x = \log_\mathrm{e} x$	$\frac{1}{x}$
$\sin x$	$\cos x$
$\sin kx$	$k \cos kx$
$\cos x$	$-\sin x$
$\cos kx$	$-k \sin kx$
$\tan x = \frac{\sin x}{\cos x}$	$\sec^2 x$
$\tan kx$	$k \sec^2 kx$

Product Rule and Quotient Rule

Product Rule:

$$y = uv \rightarrow \frac{dy}{dx} = v\frac{du}{dx} + u\frac{dv}{dx}$$

Quotient Rule:

$$y = \frac{u}{v} \rightarrow \frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

Product Rule and Quotient Rule
Examples

(i) Differentiate :

$$y = x\sin(2x)$$

(ii) Differentiate:

$$y = \frac{3x - 7}{2x + 7}$$

Stop. Try some examples:

C5

The Chain Rule by Example

(i) Differentiate :

$$y = (x^2 - 4x)^{\frac{1}{2}}$$

Set: $$u = x^2 - 4x$$

The differential is then: $$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

Stop. Try some examples:

C3