Worked Example

Consider the following multiple state model in which $S(t)$, the state occupied at time $t$ by a life initially aged $x$, is assumed to follow a continuous time Markov Jump Process.

Let $\mu_{x+t}^{ij}$ denote the force of transition at age $x+t$ ($t \geq 0$) from state $i$ to state $j$, and let $p_{x:t}^{ij} = p[S(t) = j | S(0) = i]$. 

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Let $\mu_{x+t}^{ij}$ denote the force of transition at age $x+t$ ($t \geq 0$) from state $i$ to state $j
derive the forward Kolmogorov equation:

\[
\frac{d}{dt}p_{x,t}^{2,1} = p_{x,t}^{2,2} \mu_{x+1}^{2,1} + p_{x,t}^{2,3} \mu_{x+1}^{3,1} + p_{x,t}^{2,1}(\mu_{x+1}^{1,2} + \mu_{x+1}^{1,4})
\]

Write down the forward Kolmogorov equations for \(p_{x,t}^{2,3}\) and \(p_{x,t}^{3,2}\).

Exercises

1. \(X_t\) is a poisson process that models the cumulative number of arrivals of insurance claims. The average number of claims is 0.83 per hour.
   
   • state the distribution for each \(X_t\).
   
   • calculate \(P[X_3 ≤ 2]\) and \(P[X_5 − X_2 < 3]\)
   
   • define the holding time and state its distribution
   
   • show that \(P[T_0 > s + t|T_0 > s] = P[T_0 > t]\)

2. A machine is in constant use. It tends to break down once a day and takes 8 hours to repair. you are modeling the machines status as a time homogeneous markov jump process \(X(t)\) with two states ”being repaired”=0 and ”working”=1. Let \(P_{i,j}(t)\) denote the probability that the process is in state \(j\) at time \(t\) given it was in state \(i\) at time 0 and suppose \(t\) is measured in days.
   
   • Draw the transition graph for the process showing the numerical values of the transition rates.
   
   • State the Kolmogorov backward and forward differential equations for the probability \(P_{0,0}(t)\).
   
   • Solve the forward differential equation above to show that
     \[
P_{0,0}(t) = \frac{1}{4} + \frac{3}{4} e^{-4t}
\]

3. The diagram below shows the transition diagram at time \(t\) for a time-inhomogeneous markov jump process.
(a) Write down the generator matrix for this model.

(b) Write down or derive the following equations in respect of the probability $p_{AC}(s, t)$:

i. The Chapman-Kolmogorov equation based on an intermediate time $u$ ($s < u < t$).

ii. The Kolmogorov forward differential equation

iii. The Kolmogorov backward differential equation

iv. The integrated Kolmogorov forward equation

v. The integrated Kolmogorov backward equation

(c) If instead the process is homogeneous and all the transition rates are set to 1 explain why

$$e_A = 0.5 + 0.5e_B$$