The rigidity of infinite graphs and crystals

Geometric rigidity theory has its origins in the theory of bar-joint linkages and Cauchy's rigidity theorem for convex simplicial polyhedra. A foundational result of G. Laman in 1970 characterises those finite simple graphs whose generic realisations in $\mathbb{R}^2$ give rigid bar-joint frameworks. Rigidity theory is also relevant in the condensed matter physics of crystals in connection with low energy phonon modes (rigid unit modes or RUMs) and displacive phase transitions. Here the bar-joint frameworks in mathematical models are infinite and crystallographic. I will talk about recent work with Derek Kitson. On the generic side, we obtain generalisations of the Cauchy and Laman theorems for non-Euclidean norms and for infinite graphs. On the crystallographic side we characterise almost periodic rigidity in terms of the RUM spectrum (or Bohr spectrum) of the crystal framework. Geometric rigidity theory is a wonderfully hybrid research area and in fact there is no known 3D counterpart to Laman's combinatorial characterisation. Infinite frameworks and their rigidity operators in particular invite methods from harmonic analysis and operator theory.