Microscopic models of highway traffic

overview, future directions, applications to DTA

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Overview of talk

- Link-flow models: queueing and ‘whole-link’
- Spatial structure
- Microscopic models and their macroscopic behaviour
- Future directions
- Thanks to UK Highways Agency for loan of data sets
Link flow modelling: urban case

- Travel time dominated by queueing
- (discrete time) $M/M/1$ model:

\[
\overline{q} = \frac{a(1 - a)}{p - a}, \quad \tau = \frac{\overline{q}}{a}
\]

- $a (< 1)$ arrival rate
- $p (> a)$ maximal service rate
- $\overline{q}$ (steady) average queue length
- $\tau$ average delay due to queue
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Steady / congested:
- Queue lengths are full determining variables
Link flow modelling: urban case

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- Unsteady and uncongested:
  - Queue lengths and flows are required
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Highway traffic: ‘whole-link’ models

General idea:

\[ \tau = f(x(t)) \]

- \( \tau \) transit time for vehicles entering link at time \( t \)
- \( x \) is number of vehicles on link at time \( t \)
Highway traffic: ‘whole-link’ models

- General idea:
  \[ \tau = f(x(t)) \]
  \( \tau \) transit time for vehicles entering link at time \( t \)
  \( x \) is number of vehicles on link at time \( t \)

- Conservation:
  \[ \dot{x}(t) = q_{\text{in}}(t) - q_{\text{out}}(t), \]
  \[ q_{\text{out}}(t + \tau(t)) = \frac{q_{\text{in}}(t)}{1 + \tau(t)} \]

- Some type of neutral delay differential equation
- Implicitly contains ‘spatial-like’ structure
Macroscopic traffic patterns

- displacement ($\times 500\text{m}$) vs time (minutes)
- colour: speed (km/h)
Car-following models

\begin{equation}
\begin{aligned}
x_n + 1 & \quad v_{n+1} \\
x_n & \quad h_n \\
x_n - 1 & \quad v_{n-1}
\end{aligned}
\end{equation}
Car-following models


\[ \dot{x}_n = v_n, \]
\[ \dot{v}_n = \alpha \{ V(h_n) - v_n \}. \]

- sensitivity \( \alpha > 0 \)
- \( V \) is \textit{optimal velocity} function
Bando model dynamics

What is stability of $h_n = h^*$, $v_n = V(h^*)$ steady states?
Bando model dynamics

- What is stability of \( h_n = h^* \), \( v_n = V(h^*) \) steady states?
Bando model dynamics

- What is stability of \( h_n = h^*, \ v_n = V(h^*) \) steady states?

- Unstable for \( \alpha < 2V'(h^*) \)

- Similar to real traffic data
Bando model dynamics

- What is stability of $h_n = h^*$, $v_n = V(h^*)$ steady states?

- Other refinements, e.g. driver reaction times
  G. Orosz, REW and B. Krauskopf, acc. PRE 2004
Equilibrium curves

- Speed vs. headway
- Speed vs. density
- Flow vs. density
What not to do: Gipps’s model (1982)

Kernel of much simulation software

Behavioural premise:

*What speed should I travel at given the behaviour of the driver in front one reaction time ago? If the vehicle in front brakes at (my estimate of) its hardest rate and one reaction time later I commence braking at my hardest rate, I must be able to avoid a collision.*

Analysis of Gipps’s model

- Equations of motion

\[ v_n(t+\tau) = \min \left[ v_n(t) + 2.5 A_n \tau \left( 1 - \frac{v_n(t)}{V_{\text{max}}^n} \right) \left( 0.025 + \frac{v_n(t)}{V_{\text{max}}^n} \right)^{1/2}, -B_n \left( \frac{\tau}{2} + \theta \right) \right] \]

\[ + \sqrt{B_n^2 \left( \frac{\tau}{2} + \theta \right)^2 + B_n \left\{ 2 \{x_{n-1}(t) - x_n(t) - S_{n-1}\} - \tau v_n(t) + \frac{v_{n-1}(t)^2}{\hat{B}_{n-1}} \right\}} \]

- Takes form: \[ v_n(t + \tau) = F(x_{n-1}(t) - x_n(t), v_n(t), v_{n-1}(t)) \]

- Uniform flow solutions satisfy \[ v^* = F(h^*, v^*, v^*) \]
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- Condition for instability:

\[ \theta < \left( \frac{1}{\hat{B}} - \frac{1}{B} \right) v^*, \quad \text{not physical!} \]
Other explanations for pattern

Possibility of interesting dynamics even when all uniform flows are stable
Stable waves, velocity vs vehicle index

- Travelling wave
- Expansion wave
- Compound wave
Continuum models

- Work with continuous density $\rho(x, t)$, velocity $v(x, t)$
- Conservation of vehicles

$$\rho_t + (\rho v)_x = 0.$$
Continuum models

- Work with continuous density $\rho(x, t)$, velocity $v(x, t)$
- Conservation of vehicles
  $$\rho_t + (\rho v)_x = 0.$$  
- Lighthill and Whitham 1950s
  $$v = V(\rho)$$
- $V$ speed-density function
Continuum models

- Work with continuous density \( \rho(x, t) \), velocity \( v(x, t) \)
- Conservation of vehicles

\[
\rho_t + (\rho v)_x = 0.
\]

- Relaxation effect

\[
v_t + vv_x = \alpha \{ V(\rho) - v \}\]
Continuum models

- Work with continuous density $\rho(x, t)$, velocity $v(x, t)$
- Conservation of vehicles

$$ \rho_t + (\rho v)_x = 0. $$

- Pressure term

$$ v_t + vv_x = \alpha \{ V(\rho) - v \} - \beta \frac{\rho x}{\rho} $$
Continuum models

- Work with continuous density $\rho(x, t)$, velocity $v(x, t)$
- Conservation of vehicles
  \[ \rho_t + (\rho v)_x = 0. \]
- Diffusion effects (Kerner-Konhäuser model 1994)
  \[ v_t + vv_x = \alpha \{ V(\rho) - v \} - \beta \frac{\rho x}{\rho} + \mu \frac{v_{xx}}{\rho} \]
Continuum models

- Work with continuous density $\rho(x, t)$, velocity $v(x, t)$
- Conservation of vehicles

$$\rho_t + (\rho v)_x = 0.$$ 

- Continuum limit of Bando model

$$v_t + vv_x = \alpha \{V(\rho) - v\} + \alpha V'(\rho) \left[ \frac{\rho x}{2\rho} - \frac{\rho x^2}{2\rho^3} + \frac{\rho xx}{6\rho^2} \right]$$

Berg, Mason and Woods (2000)
Travelling wave analysis

- Phase plane \((X, X')\) problem

\[
X' = Y, \\
Y' = h_1(X) \left[ \frac{q^2 Y}{X^3} + \frac{\alpha}{X} \{ Q(X) - (q + cX) \} \right] + Y h_2(X, Y).
\]

where

\[
h_1(X) = -\frac{6X^2}{\alpha V'(X)}, \quad \text{and} \quad h_2(X, Y) = -3X + \frac{3Y}{X}
\]
Travelling wave analysis

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Parametrised by \(c(\rho_-, \rho_+)\) and \(q(\rho_-, \rho_+)\)

Waves are connections from \((\rho_-, 0)\) to \((\rho_+, 0)\)
Travelling wave phase diagram

REW and P. Berg, Traffic and Granular Flow ’01
Philosophy

TO DATE (WRONG)
- Write down “plausibility models”
- Limited fit of models at macroscopic level
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IN FUTURE
- Fit models to microscopic data
- Use emergent macroscopic dynamics for predictions
M42 loop infrastructure

- Loop pairs positioned every 100m
- Measure lane, time, speed, vehicle length
- Results usually time-averaged (1 minute)
- Also individual vehicle data (IVD) sets
Relative motion and lane changes are clear

Seeking collaborators to work on this data set