Aerofoil Optimisation Using CST Parameterisation in $SU^2$

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Advanced Aero Concepts, Design and Operations

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Outline

- Methods and Key Concepts
- Alternative Parameterisations in $SU^2$
- Preliminary Results
- Conclusion
- Future Work
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Class-Shape-Transformation Method

\[ \zeta(\phi) = C_{N_2}^{N_1}(\phi)S(\phi) + \phi \Delta \zeta_{te} \]

Class Function

\[ C_{N_2}^{N_1} = \phi^{N_1}(1 - \phi)^{N_2} \]

Shape function

\[ S(\phi) = \sum_{i=0}^{n} A_i S_i \]
Class-Shape-Transformation Method

- Series of component shape functions
Class-Shape-Transformation Method
Surface Modified through choice of weights

\[ S(\phi) = \sum_{i=0}^{n} A_i S_i \]
Sensitivity Analysis

Adjoint Method

- Gradients required for optimisation
- Finite differences commonly used
- Cost proportional to number of design variables
- Cost can be unacceptable for high fidelity models
- Adjoint method provides an efficient alternative
- Independent of number of design variables
Developed by Stanford University
Open source
Freely-available
Primarily focused on aerodynamic shape optimisation
Incorporates Adjoint method for efficient gradient evaluation
Gradients made readily accessible
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Incorporating \( \text{CST} \) into \( SU^2 \)

\[
\begin{bmatrix}
\frac{\partial f}{\partial A_1} \\
\frac{\partial f}{\partial A_2} \\
\vdots \\
\frac{\partial f}{\partial A_n}
\end{bmatrix} = \begin{bmatrix}
\frac{\partial x_1}{\partial A_1} & \cdots & \frac{\partial x_m}{\partial A_1} \\
\vdots & \ddots & \vdots \\
\frac{\partial x_1}{\partial A_n} & \cdots & \frac{\partial x_m}{\partial A_n}
\end{bmatrix} \begin{bmatrix}
\frac{\partial f}{\partial x_1} \\
\frac{\partial f}{\partial x_2} \\
\vdots \\
\frac{\partial f}{\partial x_m}
\end{bmatrix}
\]

- \( \frac{\partial f}{\partial A_i} \) - Gradient 
  \[ i = 1, \ldots, n; \quad j = 1, \ldots, m \]
- \( \frac{\partial f}{\partial x_j} \) - Surface Sensitivities
- \( \frac{\partial x_j}{\partial A_i} \) - Geometric Sensitivities
Incorporating \textit{CST} into \textit{SU}^2

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\end{bmatrix}
\begin{bmatrix}
\frac{\partial f}{\partial x_1} \\
\frac{\partial f}{\partial x_2} \\
\vdots \\
\frac{\partial f}{\partial x_m}
\end{bmatrix}
\]

- \(\frac{\partial f}{\partial A_i}\) - Gradient \(i=1, \ldots, n\)
- \(\frac{\partial f}{\partial x_j}\) - Surface Sensitivities \(j=1, \ldots, m\)
- \(\frac{\partial x_j}{\partial A_i}\) - Geometric Sensitivities
Incorporating \textit{CST} into \textit{SU}^2

\[
\frac{\partial f}{\partial x_j} - \text{Surface Sensitivities}
\]

Adjoint Surface Sensitivities

Sensitivity

\((x/c)\)

-0.8 -0.6 -0.4 -0.2 0 0.2 0.4 0.6 0.8 1

0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1

lower upper
Incorporating \textit{CST} into \textit{SU}^2

\[
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\end{bmatrix}
\]

- \( \frac{\partial f}{\partial A_i} \) - Gradient
- \( \frac{\partial f}{\partial x_j} \) - Surface Sensitivities
- \( \frac{\partial x_j}{\partial A_i} \) - Geometric Sensitivities

\( i=1,\ldots,n; \ j=1,\ldots,m \)
Incorporating \textit{CST} into $SU^2$

\[ \frac{\partial x_j}{\partial A_i} - \text{Geometric Sensitivities} \]

\[
\left( \frac{dx}{dA} \right)_{j,i} = \left( \frac{\partial x_j}{\partial A_i} n_x + \frac{\partial y_j}{\partial A_i} n_y + \frac{\partial z_j}{\partial A_i} n_z \right)
\]
Incorporating CST into $SU^2$

\[
\begin{bmatrix}
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\vdots \\
\frac{\partial f}{\partial A_n}
\end{bmatrix}
= 
\begin{bmatrix}
\frac{\partial A_1}{\partial x_1} & \cdots & \frac{\partial A_1}{\partial x_m} \\
\vdots & \ddots & \vdots \\
\frac{\partial A_n}{\partial x_1} & \cdots & \frac{\partial A_n}{\partial x_m}
\end{bmatrix}
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\vdots \\
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\end{bmatrix}
\]

- $\frac{\partial f}{\partial A_i}$ - Gradient
- $\frac{\partial f}{\partial x_j}$ - Surface Sensitivities
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\[i=1,\ldots,n; \quad j=1,\ldots,m\]
Outline Methodology CST in $SU^2$ Results Conclusion Future Work

Gradient Comparison

\[ \frac{\partial f}{\partial A_i} \] - Gradient

Drag Gradients

Design Variable

Gradient

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Initial Conditions for NACA0012 optimisation:

- $M_\infty = 0.8$
- $\alpha = 1.25^\circ$
- $f = \text{min}(C_d)$
- $C_l > 0.33$
- $C_m > 0.034$
- $nDV = 8$
Inviscid Aerofoil Optimisation

(a) Initial

(b) Final
Drag Convergence

Design Cycles

Drag Coefficient, $C_d$

Design Cycles

Drag Coefficient, $C_d$

Hicks–Henne
CST

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20/26
RAE2822 optimisation:

- $M_\infty = 0.729$
- $\alpha = 2.31^\circ$
- $f = \min(C_d)$
- Reynolds n. = $6 \times 10^6$
- $nDV=8$
- initial wall spacing = $1.0 \times 10^{-5}$
- Total elements = 22842
Viscous Aerofoil Optimisation

(a) Initial

(b) Final

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Drag Convergence

Design Cycles vs. Drag Coefficient, $C_d$
Conclusion

- *CST* method introduced into $SU^2$
- Made use of Adjoint method for efficient and robust gradient evaluation
- Method seen to reduce objective function in both inviscid and viscous cases
The implementation presented here is to be extended to include parameters of a CAD model and hence form a CAD based optimisation problem.
ARE THERE ANY QUESTIONS?