Project title: Tensor Triangular Geometry for functor calculus

Supervisor(s):	**Scott Balchin, Niall Taggart**
Helpful existing knowledge:	**algebra, topology**

Project Description:

Functors can be viewed as an abstraction, or *categorification*, of functions: they transform one mathematical object into another, much as a function maps elements of one set to elements of another. *Functor calculus* extends this analogy by developing a categorical version of Taylor's theorem from classical analysis. It constructs a sequence of "polynomial" approximations, with the precise meaning of "polynomial" depending on the context. This analogy with Taylor expansion has become a central idea in modern algebraic topology, influencing a wide range of research directions.

A powerful way to understand a functor calculus is to take one step further in abstraction and study the *category of all polynomial functors* rather than individual approximations. These categories often admit elegant descriptions that reveal deep structural information about the calculus itself.

In a different but related direction, *tensor-triangular geometry* studies categories that are both triangulated and equipped with a compatible symmetric monoidal structure. This theory can be viewed as an abstraction of algebraic geometry and commutative ring theory: the operations in a tensor-triangulated category resemble the addition and multiplication of a commutative ring. The key examples for this project arise from categories of polynomial functors, but tensor-triangulated structures also appear throughout mathematics, including in the stable homotopy category and the derived category of a commutative ring.

The strength of tensor-triangular geometry lies in its ability to reconstruct a category from remarkably minimal information. Recent groundbreaking work by Arone, Barthel, Heard, and Sanders computed this minimal data for certain classes of polynomial functors. The primary goal of this PhD project is to build on their results by understanding how these minimal pieces can be assembled to reconstruct the full category of polynomial functors.

A secondary aim of the project could be to preform analogous computations for various functor calculi. In these cases, the categories are not rigidly compactly generated, and the usual methods of tensor-triangular geometry do not apply. Recent work of Barrero, Barthel, Pol, Strickland and Williamson has shown that it is still possible to produce tensor-triangular classifications in the non-rigid setting.

The student should have attended courses on both topology and algebra. This project will require the student to become familiar with the abstract language of infinity categories, stable homotopy theory and tensor-triangular geometry.

Useful references

Arone, G., Barthel, T., Heard, D., & Sanders, B., "The spectrum of excisive functors." *Inventiones mathematicae*. (2025).

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Balmer, P., “A guide to tensor-triangular classification.” *Handbook of Homotopy Theory* (2019).

Barnes, D., and Roitzheim, C. “Foundations of stable homotopy theory”. *Vol. 185. Cambridge University Press* (2020).

Barrero, M., Barthel, T., Pol, L., Strickland, N., & Williamson, J. “Global representation theory: Homological foundations.” https://arxiv.org/abs/2505.21449 (2025).

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Cnossen, B., “Stable homotopy theory and higher algebra” *available from the authors website https://sites.google.com/view/bastiaan-cnossen/home* (2025).