A Locally Optimal Test for No Unit Root in Cross-Sectionally Dependent Panel Data [1]

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Abstract

This paper develops a simple test for the null hypothesis of no unit root for panel data with cross-sectional dependence in the form of a common factor in the disturbance. We do not estimate the common factor but mop-up its effect by employing the same method as the one proposed in Pesaran (2007) in the unit root testing context. Our test is basically the same as the Kwiatkowski et al. (1992) test with the regression augmented by cross-sectional average of the observations, and hence, we call it the augmented KPSS test. We also develop a Lagrange multiplier (LM) test allowing for cross-sectional dependence and compare it with the augmented KPSS test under the null of no unit root, under the local alternative and under the fixed alternative, and discuss the differences between these two tests. We show that the augmented KPSS test is asymptotically optimal in the sense that the two tests have the same asymptotic local power, although the optimality of the augmented KPSS test is not guaranteed under a wide range of the fixed alternative.

JEL classification: C12, C33

Key words: KPSS test, unit root, cross-sectional dependence, LM test; locally best test

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1. Introduction

Since the beginning of the 1990s, much theoretical and empirical econometrics literature was devoted to testing unit root and stationarity in panel data with a large T (time dimension) and a large N (cross-section dimension). The main motive for applying unit root and stationarity tests to panel data is to improve the power of the tests relative to their univariate counterparts. This was supported by the ensuing applications and simulations. The early theoretical contributions were made from the mid-1990s to the early 2000s under the assumption that the cross-sectional units are independent or at least not cross-sectionally correlated. Banerjee (1999), Baltagi and Kao (2000), and Baltagi (2001) provide comprehensive surveys on the first generation panel tests.

However, in most empirical applications, this assumption is erroneous. O'Connell (1998) was the first to show via simulation that the panel tests are considerably distorted when the independence assumption is violated. Banerjee, Marcellino and Osbat (2001, 2004) argued against the use of panel unit root tests due to this problem. Therefore, it became imperative to develop panel tests that take the possibility of cross-sectional dependence into account. This led, recently, to a flurry of papers accounting for cross-sectional dependence in different forms or to the arrival of second generation panel unit root tests. The most noticeable proposals in this area are by Chang (2004), Phillips and Sul (2003), Bai and Ng (2004), Moon and Perron (2004), Choi and Chue (2007), and Pesaran (2007) for unit root panel tests. For panel stationarity tests, the only contributions thus far are by Bai and Ng (2005) and Harris, Leybourne and McCabe (2005), both of which corrected for cross-sectional dependence by using the principal component analysis proposed by Bai and Ng (2004).

In this paper, we focus on a test for the null hypothesis that there is no unit root in cross-sectionally dependent panel data against the alternative of the existence of unit roots. To deal with cross-sectional dependence, we adapt the Pesaran (2007) approach to the panel stationarity test of Hadri (2000) due to its conceptual simplicity. Our test is basically the same as the Kwiatkowski et al. (1992) test (KPSS test), and therefore, we call it the augmented KPSS test. We also derive a Lagrange multiplier (LM) test, which is known to be locally optimal under the assumption of normality. We show that these two tests have the same asymptotic property under the null of no unit root and under the local alternative. This implies that the augmented KPSS test is asymptotically locally optimal. Since it is much easier to construct the augmented KPSS test statistic than the LM test statistic while both tests have the same asymptotic optimality, our test is useful in practical analysis.

The paper is organized as follows. Section 2 sets up the model and assumptions, and defines the augmented test statistic. We also develop the LM test allowing for cross-sectional dependence. Section 3 is devoted to the comparison of our augmented KPSS test under restrictive assumptions with the LM test under the null of no unit root, under the local alternative and under the fixed alternative. We show that the limiting null distribution of the augmented KPSS test is the same as that of Hadri's (2000) test. In Section 4, we examine whether our theoretical result is valid in finite samples via simple Monte Carlo simulations. Section 5 gives concluding remarks. All the proofs are relegated to the Appendix.

We now give a summary on the notations. We define $M_A = I_T - A(A'A)^{-1}A'$ for a full column rank matrix A. The symbols $\overset{p\,(N,T)}{\longrightarrow}$ and $\overset{(N,T)}{\Longrightarrow}$ imply joint convergence in probability and joint weak convergence, respectively, when both N and T approach infinity simultaneously, while $\overset{T}{\Longrightarrow}$ and $\overset{N}{\Longrightarrow}$ imply weak convergence when only T or N approaches infinity.

2. Model and Test Statistics

2.1. Model and assumptions

Let us consider the following model:

$$y_{it} = z_t'\delta_i + r_{it} + u_{it}, \quad r_{it} = r_{it-1} + v_{it}, \quad u_{it} = f_t\gamma_i + \varepsilon_{it} \tag{1}$$

for $i = 1, \cdots, N$ and $t = 1, \cdots, T$, where z_t is deterministic and $r_{i0} = 0$ for all i. The commonly used specification of z_t in the literature is either $z_t = z_t^\mu = 1$ or $z_t = z_t^\tau = [1, t]'$. In this paper, we consider these two cases. Accordingly, we define $\delta_i = \alpha_i$ when $z = 1$

and $\delta_i = [\alpha_i, \beta_i]'$ when $z = [1, t]'$. In model (1), $z_t'\delta_i$ is the individual effect while f_t is the one-dimensional unobserved common factor, γ_i is the loading factor, and ε_{it} is the individual-specific (idiosyncratic) error.

By stacking y_{it} with respect to t, model (1) can be expressed as

$$\begin{bmatrix} y_{i1} \\ y_{i2} \\ \vdots \\ y_{iT} \end{bmatrix} = \begin{bmatrix} z_1' \\ z_2' \\ \vdots \\ z_T' \end{bmatrix} \delta_i + \begin{bmatrix} r_{i1} \\ r_{i2} \\ \vdots \\ r_{iT} \end{bmatrix} + \begin{bmatrix} f_1 \\ f_2 \\ \vdots \\ f_T \end{bmatrix} \gamma_i + \begin{bmatrix} \varepsilon_{i1} \\ \varepsilon_{i2} \\ \vdots \\ \varepsilon_{iT} \end{bmatrix},$$

$$\begin{bmatrix} r_{i1} \\ r_{i2} \\ \vdots \\ r_{iT} \end{bmatrix} = \begin{bmatrix} 1 & & & 0 \\ 1 & 1 & & \\ \vdots & \vdots & \ddots & \\ 1 & 1 & \cdots & 1 \end{bmatrix} \begin{bmatrix} v_{i1} \\ v_{i2} \\ \vdots \\ v_{iT} \end{bmatrix},$$

or

$$\begin{aligned} \mathbf{y}_i &= Z\delta_i + \mathbf{r}_i + \mathbf{f}\gamma_i + \varepsilon_i \qquad (2) \\ &= Z\delta_i + L\mathbf{v}_i + \mathbf{f}\gamma_i + \varepsilon_i, \end{aligned}$$

where $Z = [\tau, \mathbf{d}]$ with $\tau = [1, 1, \cdots, 1]'$ and $\mathbf{d} = [1, 2, \cdots, T]'$ being $T \times 1$ vectors, L is a $T \times T$ matrix with ones on the main diagonal and everywhere below it. Further, we have

$$\begin{bmatrix} \mathbf{y}_1 \\ \mathbf{y}_2 \\ \vdots \\ \mathbf{y}_N \end{bmatrix} = \begin{bmatrix} Z & & & \\ & Z & & \\ & & \ddots & \\ & & & Z \end{bmatrix} \begin{bmatrix} \delta_1 \\ \delta_2 \\ \vdots \\ \delta_N \end{bmatrix} + \begin{bmatrix} L & & & \\ & L & & \\ & & \ddots & \\ & & & L \end{bmatrix} \begin{bmatrix} \mathbf{v}_1 \\ \mathbf{v}_2 \\ \vdots \\ \mathbf{v}_N \end{bmatrix} + \begin{bmatrix} \mathbf{f}\gamma_1 \\ \mathbf{f}\gamma_2 \\ \vdots \\ \mathbf{f}\gamma_N \end{bmatrix} + \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_N \end{bmatrix}$$

or

$$\begin{aligned} \mathbf{y} &= (I_N \otimes Z)\delta + \mathbf{r} + (\gamma \otimes \mathbf{f}) + \varepsilon \qquad (3) \\ &= (I_N \otimes Z)\delta + (I_N \otimes L)\mathbf{v} + (\gamma \otimes \mathbf{f}) + \varepsilon. \end{aligned}$$

In this paper, we make the following assumption.

Assumption 1 *(i) The stochastic processes $\{\varepsilon_{it}\}$, $\{f_t\}$, and $\{v_{it}\}$ are independent and*

$$\varepsilon_{it} \sim i.i.d.N(0, \sigma_\varepsilon^2), \quad f_t \sim i.i.d.N(0, \sigma_f^2), \quad v_{it} \sim i.i.d.N(0, \sigma_v^2) \quad \textit{with known variances.}$$

(ii) There exist real numbers M_1, $\underline{M}$, and $\overline{M}$ such that $|\gamma_i| < M_1 < \infty$ for all i and $0 < \underline{M} < |\bar{\gamma}| < \overline{M} < \infty$ for all N, where $\bar{\gamma} = N^{-1}\sum_{i=1}^{N}\gamma_i$.

The assumption of normality with homoskedasticity in (i) is required to derive the LM test and to discuss the optimal property of the tests. The variances σ_ε^2, σ_f^2, and σ_v^2 are assumed to be known in order to make the theoretical investigation as simple as possible. The unknown case will be discussed later. (ii) implies that each individual is possibly affected by the common factor with the finite weight γ_i and that the absolute value of the average of γ_i is bounded away from 0 and above both in finite samples and in asymptotics. The latter property is important in order to eliminate the common factor effect from the regression. See also Pesaran (2007).

We consider a test for the null hypothesis of no unit root component against the alternative of the existence of unit roots for model (1). Since all the innovations are homoskedastic, the testing problem is given by

$$H_0: \ \rho = 0 \quad \text{vs.} \quad H_1: \rho > \ 0 \tag{4}$$

where $\rho = \sigma_v^2/\sigma_\varepsilon^2$ is the signal-to-noise ratio. Under H_0, all r_{it}s become equal to zero and thus do not have unit root components, unlike under H_1.

2.2. A simple stationarity test

Panel stationarity tests have already been proposed by Hadri (2000) and Shin and Snell (2006) for cross-sectionally independent data, and we extend Hadri's test to the cross-sectionally dependent case. Hadri (2000) showed that if there is no cross-sectional dependence in a model, we can construct the LM test using the regression residuals of y_{it} on z_t in the same way as KPSS (1992), and that the limiting distribution of the standardized LM test statistic is standard normal under the null hypothesis. However, it can be shown that Hadri's (2000) test depends on nuisance parameters even asymptotically if there exits cross-sectional dependence; we then need to develop a stationarity test that takes into account cross-sectional dependence.

In order to eliminate the effect of the common factor from the test statistic, we make use of the simple method proposed by Pesaran (2007), which develops panel unit root tests with cross-sectional dependence. As in Pesaran (2007), we first take a cross-sectional average of the model:

$$\bar{y}_t = z_t'\bar{\delta} + \bar{r}_t + f_t\bar{\gamma} + \bar{\varepsilon}_t, \tag{5}$$

where $\bar{y}_t = N^{-1}\sum_{i=1}^N y_{it}$, $\bar{\delta} = N^{-1}\sum_{i=1}^N \delta_i$, $\bar{r}_t = N^{-1}\sum_{i=1}^N r_{it}$, $\bar{\gamma} = N^{-1}\sum_{i=1}^N \gamma_i$, and $\bar{\varepsilon}_t = N^{-1}\sum_{i=1}^N \varepsilon_{it}$. Since $\bar{\gamma} \neq 0$ by assumption, we can solve equation (5) with respect to f_t as

$$f_t = \frac{1}{\bar{\gamma}}\left(\bar{y}_t - z_t'\bar{\delta} - \bar{r}_t - \bar{\varepsilon}_t\right).$$

By inserting this solution of f_t into model (1), we obtain the following augmented regression model:

$$y_{it} = z_t'\tilde{\delta}_i + \tilde{\gamma}_i\bar{y}_t + \epsilon_{it}, \tag{6}$$

where $\tilde{\delta}_i = \delta_i - \tilde{\gamma}_i\bar{\delta}$, $\tilde{\gamma}_i = \gamma_i/\bar{\gamma}$, and $\epsilon_{it} = r_{it} - \tilde{\gamma}_i\bar{r}_t + \varepsilon_{it} - \tilde{\gamma}_i\bar{\varepsilon}_t$. Based on (6), we propose to regress y_{it} on z_t and $\bar{y}_t$ for each i, and construct the test statistic in the same way as Hadri (2000). That is,

$$Z_A = \frac{\sqrt{N}(\overline{ST} - \xi)}{\zeta}, \tag{7}$$

where $\overline{ST} = N^{-1}\sum_{i=1}^N ST_i$ with $ST_i = (\sigma_\varepsilon^2 T^2)^{-1}\mathbf{y}_i' M_w L' L M_w \mathbf{y}_i$ and

$$\begin{cases} \xi = \xi_\mu = \frac{1}{6}, \quad \zeta^2 = \zeta_\mu^2 = \frac{1}{45} & \text{when } \ z_t = z_t^\mu = 1, \\ \xi = \xi_\tau = \frac{1}{15}, \quad \zeta^2 = \zeta_\tau^2 = \frac{11}{6300} & \text{when } \ z_t = z_t^\tau = [1, t]'. \end{cases}$$

Note that ST_i can also be expressed as

$$ST_i = \frac{1}{\sigma_\varepsilon^2 T^2}\sum_{t=1}^T (S_{it}^w)^2 \quad \text{where} \quad S_{it}^w = \sum_{s=1}^t \hat{\epsilon}_{is}$$

with $\hat{\epsilon}_{it}$ obtained for each i by regressing y_{it} on $w_t = [z_t', \bar{y}_t]'$ for $t = 1, \cdots, T$.

From (7), we can see that $\overline{ST}$ is the average of the KPSS test statistic across i and Z_A corresponds to its normalized version. We call Z_A the augmented KPSS test statistic.

2.3. An LM test for panel stationarity

Although the augmented KPSS test is easy to implement, we do not know whether it has an optimal property. Since the LM test is known to be a locally best invariant test under the assumption of normality as shown by Tanaka (1996), we derive the LM test, and then, in the later section, compare it with the augmented KPSS test.

Under Assumption 1, the log-likelihood function of $\mathbf{y}$, denoted by ℓ, is expressed as

$$\ell = const - \frac{1}{2}\log|\Omega| - \frac{1}{2}\{\mathbf{y} - (I_N \otimes Z)\delta\}'\Omega^{-1}\{\mathbf{y} - (I_N \otimes Z)\delta\},$$

where $\Omega = Var(\mathbf{y}) = \rho\left(\sigma_\varepsilon^2 I_N \otimes LL'\right) + A \otimes I_T$ with $A = \sigma_f^2\gamma\gamma' + \sigma_\varepsilon^2 I_N$. The partial derivative of ℓ with respect to ρ is given by

$$\frac{\partial \ell}{\partial \rho} = const + \frac{1}{2}\{\mathbf{y} - (I_N \otimes Z)\delta\}'\Omega^{-1}\frac{\partial \Omega}{\partial \rho}\Omega^{-1}\{\mathbf{y} - (I_N \otimes Z)\delta\}. \tag{8}$$

Noting that

$$\Omega|_{H_0} = A \otimes I_T \quad \text{and} \quad \left.\frac{\partial \Omega}{\partial \rho}\right|_{H_0} = \sigma_\varepsilon^2 I_N \otimes LL', \tag{9}$$

the maximum likelihood estimator (MLE) of δ under H_0 is given by

$$\begin{aligned}\hat{\delta} &= \left[(I_N \otimes Z')\left.\Omega^{-1}\right|_{H_0}(I_N \otimes Z)\right]^{-1}(I_N \otimes Z')\left.\Omega^{-1}\right|_{H_0}\mathbf{y} \\ &= [I_N \otimes (Z'Z)^{-1}Z']\mathbf{y}. \end{aligned} \tag{10}$$

Thus, the MLE of δ under H_0 is the same as the OLS estimator. By evaluating (8) under the null hypothesis using (9) and (10), the LM test statistic is given by

$$\begin{aligned}\overline{LM} &= \frac{1}{NT^2}\{\mathbf{y} - (I_N \otimes Z)\hat{\delta}\}'(A^{-1} \otimes I_T)(\sigma_\varepsilon^2 I_N \otimes LL')(A^{-1} \otimes I_T)\{\mathbf{y} - (I_N \otimes Z)\hat{\delta}\} \\ &= \frac{1}{NT^2}\mathbf{y}'(\sigma_\varepsilon^2 A^{-2} \otimes M_z LL' M_z)\mathbf{y}.\end{aligned}$$

Then, the normalized version of the LM test statistic is given by

$$Z_{LM} = \frac{\sqrt{N}\left(\overline{LM} - \xi\right)}{\zeta}, \tag{11}$$

where ξ and ζ are the same as in Z_A.

3. Limiting Distributions of the Test Statistics

In this section, we compare the augmented KPSS test with the LM test. Note that the LM test is known to be a locally best invariant test under Assumption 1. Because there is no one-to-one transformation between Z_A and Z_{LM}, the augmented KPSS test does not have local optimality in finite samples. As such, we now focus on whether the KPSS test is asymptotically locally optimal or not.

In order to investigate the asymptotic local optimality of the augmented KPSS test, we compare it with the LM test statistic under the null hypothesis, under the local alternative and under the fixed alternative. We first give the limiting distributions of the two test statistics under the null hypothesis.

Theorem 1 *Suppose that Assumption 1 holds. Under H_0, as N and T approach infinity simultaneously with $N/T \to 0$, the augmented KPSS and LM test statistics have a limiting standard normal distribution for both cases of $z_t = 1$ and $z_t = [1, t]'$. That is, $Z_A, Z_{LM} \overset{(N,T)}{\Longrightarrow} N(0, 1)$.*

Note that the rejection regions of both Z_A and Z_{LM} are the right-hand tails as in Hadri's (2000) test. Theorem 1 shows that Pesaran's (2007) method works well to eliminate cross-sectional dependence for testing the null hypothesis of stationarity. We also note that the condition that $N/T \to 0$ as N and T approach infinity, means that the tests are suitable for panels where T is larger than N.

We now investigate the asymptotic property of the test statistics under the local alternative, which is expressed as

$$H_1^{\ell} : \ \rho = \frac{c^2}{\sqrt{N}T^2} \quad \text{, where } c \text{ is some constant.}$$

Note that for a single time series analysis, the local alternative is given by $\rho = c^2/T^2$. Since the sum of ST_i is normalized by $\sqrt{N}$ as in Z_A, the local alternative for panel stationarity tests becomes $\rho = c^2/(\sqrt{N}T^2)$.

Theorem 2 *Suppose that Assumption 1 holds. Under H_1^{ℓ}, as N and T approach infinity simultaneously with $N/T \to 0$, the augmented KPSS and LM test statistics have the same* *limiting distribution given by*

$$Z_A,\ Z_{LM} \overset{(N,T)}{\Longrightarrow} N(0,1) + \frac{c^2}{\zeta} E\left[\int_0^1 F_i^v(r)^2 dr\right],$$

where $F_i^v(r) = \int_0^r B_i^v(s)ds - \int_0^r z(s)'ds \left(\int_0^1 z(s)z(s)'ds\right)^{-1} \int_0^1 z(s)B_i^v(s)ds$ *with* $B_i^v(r)$ *being independent Brownian motions,* $z(r) = 1$ *and* $E[\int_0^1 F_i^v(r)^2 dr] = \sqrt{1/180}$ *when* $z_t = 1$*, and* $z(r) = [1, r]'$ *and* $E[\int_0^1 F_i^v(r)^2 dr] = \sqrt{11/25200}$ *when* $z_t = [1, t]'$.

This result implies that both the augmented KPSS and extended LM test statistics have the same asymptotic local distribution. Since the LM test is locally best invariant, we can see that the augmented KPSS test has the same asymptotic local optimality.

We can also deduce from Theorem 2 that both tests are more powerful when only a constant is included in the regression than in the trending case, much like the univariate KPSS test, because $1/90 > 11/12600$.

We finally investigate the asymptotic property of the test statistics under the fixed alternative H_1. The following theorem gives the difference in the powers of the two tests when the alternative is not local but far away from $\rho = 0$.

Theorem 3 *Suppose that Assumption 1 holds. Under* H_1*, as* N *and* T *approach infinity simultaneously with* $N/T \to 0$*,*

$$\frac{1}{\sqrt{N}T^2} Z_A \overset{(N,T)}{\Longrightarrow} \frac{\rho}{\zeta} E_{vi}\left[\int_0^1 G_i^v(r)^2 dr\right], \quad and \quad \frac{1}{\sqrt{N}T^2} Z_{LM} \overset{p\ (N,T)}{\longrightarrow} \frac{\rho}{\zeta} E\left[\int_0^1 F_i^v(r)^2 dr\right],$$

where $G_i^v(r) = \int_0^r B_i^v(s)ds - \int_0^r z_2'(s)ds(\int_0^1 z_2(s)z_2'(s)ds)^{-1} \int_0^1 z_2(s)B_i^v(s)ds$ *with* $z_2(r) = [z'(r), \underline{B}^v(r)]'$*,* $\underline{B}^v(r)$ *is a standard Brownian motion independent of* $B_i^v(r)$*, and* E_{vi} *denotes the expectation operator with respect to* $B_i^v(r)$*.*

Note that since $G_i^v(r)$ depends on $B_i^v(r)$ and $\underline{B}^v(r)$, which are independent, we can see that $E_{vi}[\int_0^1 G_i^v(r)^2 dr]$ still depends on $\underline{B}^v(\cdot)$ and is thus stochastic, while $E[\int_0^1 F_i^v(r)^2 dr]$ is deterministic. This is an interesting result because when the asymptotic local powers are the same for the two tests, it is often the case that they also have the same limiting distribution under the fixed alternative. In our situation, the two tests have the same local asymptotic power from Theorem 2 but the powers are different under the fixed alternative from Theorem 3. This implies that although the two tests are locally optimal, they are not equivalent in a wide range under the alternative.

Finally, we discuss the case where the variances are unknown. In this case, we can estimate σ_ε^2 consistently under H_0 by $(NT)^{-1}\sum_{i=1}^N\sum_{t=1}^T\hat{\varepsilon}_{it}^2$, where $\hat{\varepsilon}_{it}^2$ is the residual from the augmented regression. Then, we can still construct Z_A in practical analysis. However, the construction of the LM test requires the knowledge of not only σ_ε^2 but also $\sigma_f^2\gamma\gamma'$ as in the definition of A, which can be obtained by the method in Bai (2003). However, since Z_A is much simpler than Z_{LM}, and Z_A is asymptotically locally optimal, the augmented KPSS test would be convenient and useful in practical analysis.

4. Finite sample property

In this section, we investigate how accurately does the asymptotic theory approximate the finite sample behavior of the augmented KPSS and LM tests. We consider the following data generating process for finite sample simulations:

$$y_{it} = z_t'\delta_i + r_{it} + f_t\gamma_i + \varepsilon_{it}, \quad f_t \sim i.i.d.N(0,1), \quad \varepsilon_{it} \sim i.i.d.N(0,1),$$
$$r_{it} = r_{it-1} + v_{it}, \quad v_{it} \sim i.i.d.N(0,\rho), \quad \begin{cases} H_0: \ \rho = 0, \\ H_1: \ \rho = 0.0001, \ 0.001, 0.01. \end{cases}$$

where $\delta_i = \alpha_i$ for the constant case while $\delta_i = [\alpha_i, \beta_i]'$ for the trend case with α_i and β_i being drawn from independent $U(0, 0.02)$, γ_i are drawn from $-1 + U(0,4)$ for the strong cross-sectional correlation case (SCC) and from $U(0, 0.02)$ for the weak cross-sectional correlation case (WCC), and α_i, β_i, and γ_i are fixed throughout the iterations. Since our purpose is to see if the asymptotic theory obtained in the previous section can approximate the finite sample behavior, we assume that the variances are known throughout the simulations. We consider all the pairs of $N = 10$, 20, 30, 50, and 100, and $T = 50$, 100, and 200. The level of significance is 0.05 and the number of replications is 10,000 in all experiments.

Table 1 shows the sizes of the tests. We can observe that the empirical size of the augmented KPSS test is close to the nominal one for any value of T for the SCC case while it is slightly undersized for the WCC case. On the other hand, the size of the LM test is close to the nominal one irrespective of N and T but it is slightly undersized for the SCC case while it is slightly oversized for the WCC case. Overall, the null distributions of the two tests seem to be well approximated by a standard normal distribution as suggested by Theorem 1 in view of the size of the tests.

Table 2 reports the powers of the tests. For given N and T, the upper, middle, and lower entries are the powers of the tests for $\rho = 0.0001$, 0.001, and 0.01, respectively. From the table, the powers of the tests become higher for larger ρ and T, although the tests have low power when T is small. We can also observe that the powers become higher for larger N. For example, the size of the augmented KPSS test for $T = 50$, SCC, and the constant case is relatively close to 0.05 for all the values of N while the empirical power when $\rho = 0.001$ is 0.145, 0.202, 0.254, 0.342, and 0.539 for $N = 10$, 20, 30, 50, and 100, respectively. Table 2 implies that the tests are consistent as proved by Theorem 3.

In order to see if the augmented KPSS test can be seen as the asymptotically locally best test indicated by Theorem 2, we calculated the size adjusted power of the tests. Figure 1 draws the power curves for selected cases. From the figure, we observe that the power of the augmented KPSS test is almost the same as that of the LM test for the constant case. When a linear trend is included, the augmented KPSS test is as powerful as the LM test when ρ is small while the former is slightly less powerful than the latter for the trend case.

As a whole, the finite sample behavior of the augmented KPSS and LM tests is well approximated by the asymptotic theory established in the previous section when N and T are of moderate size.

5. Conclusion

In this paper we extended Hadri's (2000) test to correct for cross-sectional dependence à la Pesaran (2007). We showed that the limiting null distribution of the augmented KPSS test is the same as that of Hadri's test that assumes cross-sectional dependence. We also derived the LM test under the assumption of cross-sectional dependence. Then, we compared these two tests and found that the augmented KPSS test is asymptotically locally optimal but it is not asymptotically equivalent to the LM test under the fixed alternative.

Although the augmented KPSS test has a local optimal property, we do not know the theoretical and finite sample property of the test when the idiosyncratic errors are serially correlated. In addition, we assumed a one-dimensional common factor in this paper but it would be worth considering multi-dimensional common factors. The modification of our test to such a general case is our ongoing research.

References

[1] Bai, J. (2003). Inference on factor models of large dimensions. *Econometrica 70*, 191-221.

[2] Bai, J. and S. Ng (2004). A panic attack on unit roots and cointegration. *Econometrica 72*, 1127-1177.

[3] Bai, J. and S. Ng (2005). A new look at panel testing of stationarity and the PPP hypothesis, D. W. K. Andrews and J. H. Stock, eds., *Identification and Inference for Econometric Models. Essays in Honor of Thomas Rothenberg*. Cambridge University Press, Cambridge.

[4] Baltagi, B. H. (2001). *Econometric Analysis of Panel Data*. Wiley, Chichester.

[5] Baltagi, B. H. and C. Kao (2000). Nonstationary panels, cointegration in panels and dynamic panels: A survey. *Advances in Econometrics 15*, 7-51.

[6] Banerjee, A. (1999). Panel data unit roots and cointegration: An overview. *Oxford Bulletin of Economics and Statistics Special Issue*, 607-29.

[7] Banerjee, A., M. Marcellino and C. Osbat (2001). Testing for PPP: Should we use panel methods? *Empirical Economics 30*, 77-91.

[8] Banerjee, A., M. Marcellino and C. Osbat (2004). Some cautions on the use of panel methods for integrated series of macroeconomic data. *Econometrics Journal 7*, 322-340.

[9] Chang, Y. (2004). Bootstrap unit root tests in panels with cross-sectional dependency. *Journal of Econometrics 110*, 261-292.

[10] Choi, I. and T. K. Chue (2007). Subsampling hypothesis tests for nonstationary panels with applications to exchange rates and stock prices. *Journal of Applied Econometrics 22*, 233-264.

[11] Chung, K. L. (1974). *A Course in Probability Theory (2nd ed.)*, Academic Press, San Diego.

[12] Kwiatkowski, D., P. C. B. Phillips, P. Schmidt and Y. Shin (1992). Testing the null hypothesis of stationarity against the alternative of a unit root. *Journal of Econometrics 54*, 159-178.

[13] Hadri, K. (2000). Testing for stationarity in heterogeneous panel data. *Econometrics Journal 3*, 148-161.

[14] Harris, D., S. Leybourne and B. McCabe (2005). Panel stationarity tests for purchasing power parity with cross-sectional dependence. *Journal of Business and Economic Statistics 23*, 395-409.

[15] Moon, R. and B. Perron (2004). Testing for unit roots in panels with dynamic factors. *Journal of Econometrics 122*, 81-126

[16] Pesaran, M. H. (2007). A simple panel unit root test in the presence of cross-section dependence. *Journal of Applied Econometrics 22*, 265-312.

[17] Phillips, P. C. B. and H. R. Moon (1999). Linear regression limit theory for nonstationary panel data. *Econometrica 67*, 1057-1111.

[18] Phillips, P. C. B. and D. Sul (2003). Dynamic panel estimation and homogeneity testing under cross section dependence. *Econometrics Journal 6*, 217-25.

[19] O'Connell, P. G. J. (1998). The overvaluation of purchasing power parity. *Journal of International Economics 44*, 1-19.

[20] Shin, Y. and A. Snell (2006). Mean group tests for stationarity in heterogeneous panels. *Econometrics Journal 9*, 123-158.

[21] Tanaka, K. (1996). *Time Series Analysis: Nonstationary and Noninvertible Distribution Theory*. Wiley, New York.

Appendix

In this appendix, we denote some constants independent of N, T, and the subscripts i and t as C, C_1, C_2, $\cdots$. To save space, we give the outline of the proof of the theorems only for the case where $z_t = [1, t]$. Details are available upon request. The proof for the level case with $z_t = 1$ proceeds in exactly the same way, and is thus omitted. We also assume that $\sigma_\varepsilon^2 = 1$ in this appendix without loss of generality because we know σ_ε^2 under Assumption 1(i).

We first express $\bar{y}_t$ in matrix form. Since $\bar{y}_t = z_t'\bar{\delta} + \bar{r}_t + f_t\bar{\gamma} + \bar{\varepsilon}_t$, we have

$$\bar{\mathbf{y}} = Z\bar{\delta} + \bar{\mathbf{r}} + \mathbf{f}\bar{\gamma} + \bar{\varepsilon}, \tag{12}$$

where, for example, $\bar{\mathbf{y}} = [\bar{y}_1', \bar{y}_2', \cdots, \bar{y}_T']'$ and the other vectors and matrices are defined similarly. Since $\bar{\gamma} \neq 0$, we have $\mathbf{f} = (\bar{\mathbf{y}} - Z\bar{\delta} - \bar{\mathbf{r}} - \bar{\varepsilon})/\bar{\gamma}$. By inserting this into (2), the model becomes

$$\mathbf{y}_i = Z(\delta_i - \tilde{\gamma}_i\bar{\delta}) + \tilde{\gamma}_i\bar{\mathbf{y}} + (\mathbf{r}_i - \tilde{\gamma}_i\bar{\mathbf{r}}) + (\varepsilon_i - \tilde{\gamma}_i\bar{\varepsilon}), \tag{13}$$

where $\tilde{\gamma}_i = \gamma_i/\bar{\gamma}$.

Let $W = [\tau, \mathbf{d}, \bar{\mathbf{y}}] = [Z, \bar{\mathbf{y}}]$ and $W^* = WQ = [Z, \bar{\mathbf{y}}^*]$, where $\bar{\mathbf{y}}^* = \bar{\mathbf{y}} - Z\bar{\delta} = \bar{\mathbf{r}} + \mathbf{f}\bar{\gamma} + \bar{\varepsilon}$,

$$Q = \begin{bmatrix} I_2 & -\bar{\delta} \\ 0 & 1 \end{bmatrix}, \quad D = \begin{bmatrix} D_\tau & 0 \\ 0 & \sqrt{T} \end{bmatrix} \quad \text{and} \quad D_\tau = \begin{bmatrix} \sqrt{T} & 0 \\ 0 & T\sqrt{T} \end{bmatrix}.$$

Because $M_w = M_{w^*}$, ST_i in the augmented KPSS test statistic can be expressed in matrix form as

$$ST_i = \frac{1}{T^2}\mathbf{y}_i' M_{w^*} L' L M_* \mathbf{y}_i.$$

Before proceeding with the proof of the theorems, we state two lemmas, which will be used in the proof repeatedly.

Lemma A.1 *Let $v_{it} \sim i.i.d.N(0, \sigma_v^2)$ for $i = 1, \cdots, N$ and $t = 1, \cdots, T$, $r_{it} = \sum_{s=1}^{t} v_{is}$, and $\bar{r}_t = N^{-1}\sum_{i=1}^{N} r_{it}$. Then,*

$$\begin{aligned}
E\left[r_{is} r_{it}\right] &= \sigma_v^2 \min(s,t), & (14)\\
E\left[\left(\sum_{s=1}^{t} r_{is}\right)^2\right] &= \frac{\sigma_v^2}{6} t(t+1)(2t+1), & (15)\\
E\left[\left(\sum_{s=1}^{t} s r_{is}\right)^2\right] &= \frac{\sigma_v^2}{30} t(t+1)(2t+1)(2t^2+2t+1), & (16)\\
E\left[\bar{r}_s \bar{r}_t\right] &= \frac{\sigma_v^2}{N} \min(s,t), & (17)\\
E\left[\left(\sum_{s=1}^{t} \bar{r}_s\right)^2\right] &= \frac{\sigma_v^2}{6N} t(t+1)(2t+1), & (18)\\
E\left[\left(\sum_{s=1}^{t} s\bar{r}_s\right)^2\right] &= \frac{\sigma_v^2}{30N} t(t+1)(2t+1)(2t^2+2t+1), & (19)
\end{aligned}$$

$$\begin{aligned}
E\left[\left(\sum_{s=1}^{t} r_{is}\right)\left(\sum_{t=1}^{T} r_{it}\right)\right] &= \frac{\sigma_v^2}{6} t(t+1)(3T-t+1), & (20)\\
E\left[\left(\sum_{s=1}^{t} r_{is}\right)\left(\sum_{t=1}^{T} t r_{it}\right)\right] &= \frac{\sigma_v^2}{24} t(t+1)(6T^2+6T-t^2-t+2), & (21)\\
E\left[\left(\sum_{t=1}^{T} r_{it}\right)\left(\sum_{t=1}^{T} t r_{it}\right)\right] &= \frac{\sigma_v^2}{24} T(T+1)(5T^2+5T+2), & (22)\\
E\left[r_{is} r_{it} r_{iu} r_{iv}\right] &= \sigma_v^4 (2st+su) \quad \textit{for} \quad s \le t \le u \le v. & (23)
\end{aligned}$$

The next lemma gives the sufficient conditions on the equivalence of the sequential limit to the joint limit. Notice that when the statistic S_{iT} weakly converges to $S_{i\infty}$ as $T \to \infty$, we can construct the probability space on which both S_{iT} and $S_{i\infty}$ exist, as discussed in Phillips and Moon (1999).

Lemma A.2 *Let S_{iT} and $S_{i\infty}$ be i.i.d. sequences across i ($i = 1, \cdots, N$) on the same probability space. Assume that $S_{i\infty}$ does not depend on N, S_{iT} is independent of $S_{j\infty}$ for $i \neq j$, and $S_{iT} \stackrel{T}{\Longrightarrow} S_{i\infty}$ as $T \to \infty$.*
(i) If (a) $E[S_{iT}] \to \mu_1 \equiv E[S_{i\infty}] < \infty$ as both N and T approach infinity, and (b) $\sup_{N,T} E[S_{iT}^2] < \infty$, then,

$$\frac{1}{N}\sum_{i=1}^{N} S_{iT} \stackrel{p\ (N,T)}{\longrightarrow} \mu_1.$$

(ii) If (a) $N^{-1/2}\sum_{i=1}^{N} S_{i\infty} \stackrel{N}{\Longrightarrow} S$ as $N \to \infty$, (b) S_{iT} does not depend on N and $\sup_T E[S_{iT}^{2+\kappa_1}] < \infty$ for some $\kappa_1 > 0$ or $E[S_{iT}^2] \to \mu_2 \equiv E[S_{i\infty}^2] < \infty$ as $T \to \infty$, (c) $\sup_T E[S_{iT}^2] < \infty$ and $E[S_{i\infty}^{2+\kappa_2}] < \infty$ for some $\kappa_2 > 0$, then,

$$\frac{1}{\sqrt{N}}\sum_{i=1}^{N} S_{iT} \stackrel{(N,T)}{\Longrightarrow} S.$$

Proof of Lemma A.2: (i) Since S_{iT} is an i.i.d. sequence, we have for any arbitrary $\varepsilon > 0$,

$$P\left(\left|\frac{1}{N}\sum_{i=1}^{N} S_{iT} - E[S_{iT}]\right| \geq \varepsilon\right) \leq \frac{1}{\varepsilon^2 N} E[(S_{iT} - E[S_{iT}])^2] \leq \frac{1}{\varepsilon^2 N}\sup_{N,T} E[S_{iT}^2] \to 0$$

by condition (b) as both N and T approach infinity. Because $E[S_{iT}] \to \mu_1$ by condition (a), we can see that $N^{-1}\sum_{i=1}^{N} S_{iT} \stackrel{p\ (N,T)}{\longrightarrow} \mu_1$.

(ii) Since S_{iT} and $S_{i\infty}$ are i.i.d. sequences, we have for any arbitrary $\varepsilon > 0$,

$$P\left(\left|\frac{1}{\sqrt{N}}\sum_{i=1}^{N} (S_{iT} - S_{i\infty})\right| \geq \varepsilon\right) \leq \frac{1}{\varepsilon^2}\left(E[S_{iT}^2] + E[S_{i\infty}^2] - 2E[S_{iT}S_{i\infty}]\right). \qquad (24)$$

If $\sup_T E[S_{it}^{2+\kappa_1}] < \infty$, then we can replace the limit and the expectation by Theorem 4.5.2. of Chung (1974), and thus, $\lim_T E[S_{iT}^2] = E[S_{i\infty}^2]$ under condition (b). On the other hand, by Hölder's inequality, we have for any arbitrary $0 < \delta < 1$,

$$E\left[|S_{iT}S_{i\infty}|^{1+\delta}\right] \leq \left(E\left[S_{iT}^2\right]\right)^{(1+\delta)/2}\left(E\left[|S_{i\infty}|^{2(1+\delta)/(1-\delta)}\right]\right)^{(1-\delta)/2}.$$

The right-hand side of the above inequality with $\delta = \kappa_2/(4+\kappa_2)$ is bounded above uniformly over T by condition (c). This implies that $\sup_T E[|S_{iT}S_{i\infty}|^{1+\kappa_2/(4+\kappa_2)}] < \infty$, and again, we can replace the limit and the expectation, so that $\lim_T E[S_{iT}S_{i\infty}] = E[S_{i\infty}^2]$. As a result, the right-hand side of (24) approaches zero as both N and T approach infinity. Combining this result with condition (a), we obtain (ii).□

Proof of Theorem 1

Because $\mathbf{r}_i$ and $\bar{\mathbf{r}}$ disappear under the null hypothesis, ST_i can be expressed in matrix form under H_0 as

$$\begin{aligned} ST_i &= \frac{1}{T^2}\mathbf{y}_i' M_{w^*} L' L M_{w^*}\mathbf{y}_i \\ &= \frac{1}{T^2}\varepsilon_i' M_{w^*} L' L M_{w^*}\varepsilon_i - \frac{2\tilde{\gamma}_i}{T^2}\bar{\varepsilon}' M_{w^*} L' L M_{w^*}\varepsilon_i + \frac{\tilde{\gamma}_i^2}{T^2}\bar{\varepsilon}' M_{w^*} L' L M_{w^*}\bar{\varepsilon} \\ &= ST_{1i} - 2\tilde{\gamma}_i ST_{2i} + \tilde{\gamma}_i^2 ST_{3i}, \quad \text{say.} \end{aligned}$$

Let $ST_{1i}^0 = T^{-2}\varepsilon_i' M_z L' L M_z \varepsilon_i$. Since Shin and Snell (2006) showed that

$$\frac{\frac{1}{\sqrt{N}}\sum_{i=1}^N (ST_{1i}^0 - \xi)}{\zeta} \overset{(N,T)}{\Longrightarrow} N(0,1),$$

it is sufficient for us to prove that

$$\frac{1}{\sqrt{N}}\sum_{i=1}^N (ST_{1i} - ST_{1i}^0) \quad \overset{p\ (N,T)}{\longrightarrow} \quad 0, \tag{25}$$

$$\frac{1}{\sqrt{N}}\sum_{i=1}^N ST_{2i} \quad \overset{p\ (N,T)}{\longrightarrow} \quad 0, \tag{26}$$

$$\frac{1}{\sqrt{N}}\sum_{i=1}^N ST_{3i} \quad \overset{p\ (N,T)}{\longrightarrow} \quad 0. \tag{27}$$

Let $J_{0i} = T^{-1}L\varepsilon_i$, $[J_1, J_2] = T^{-1}LW^*D^{-1} = [T^{-1}LZD_\tau^{-1}, T^{-3/2}L\bar{\mathbf{y}}^*]$, $[J_{3i}', J_{4i}']' = D^{-1}W^{*\prime}\varepsilon_i = [(D_\tau^{-1}Z'\varepsilon_i)', (T^{-1/2}\bar{\mathbf{y}}^{*\prime}\varepsilon_i)']'$, $K = [[K_{ij}]] = D^{-1}W^{*\prime}W^*D^{-1}$, and $K^{-1} = [[K^{ij}]]$ for $i, j = 1, 2$. Then, we have

$$\begin{aligned} \frac{1}{T}LM_{w^*}\varepsilon_i &= \frac{1}{T}L\varepsilon_i - \frac{1}{T}LW^*D^{-1}\left(D^{-1}W^{*\prime}W^*D^{-1}\right)^{-1}D^{-1}W^{*\prime}\varepsilon_i \\ &= (J_{0i} - J_1K^{11}J_{3i}) - \{J_2K^{21}J_{3i} + (J_1K^{12} + J_2K^{22})J_{4i}\}. \end{aligned} \tag{28}$$

Similarly, by letting $\bar{J}_0 = N^{-1}\sum_{i=1}^N J_{0i}$, $\bar{J}_3 = N^{-1}\sum_{i=1}^N J_{3i}$, and $\bar{J}_4 = N^{-1}\sum_{i=1}^N J_{4i}$, we can see that

$$\frac{1}{T}LM_{w^*}\bar{\varepsilon}_i = \bar{J}_0 - (J_1K^{11} + J_2K^{21})\bar{J}_3 - (J_1K^{12} + J_2K^{22})\bar{J}_4. \tag{29}$$

We first prove (25). Using expression (28), ST_{1i} can be decomposed into

$$\begin{aligned} ST_{1i} &= (J_{0i} - J_1K^{11}J_{3i})'(J_{0i} - J_1K^{11}J_{3i}) \\ &\quad -2(J_{0i} - J_1K^{11}J_{3i})'\{J_2K^{21}J_{3i} + (J_1K^{12} + J_2K^{22})J_{4i}\} \\ &\quad +\{J_2K^{21}J_{3i} + (J_1K^{12} + J_2K^{22})J_{4i}\}'\{J_2K^{21}J_{3i} + (J_1K^{12} + J_2K^{22})J_{4i}\} \\ &= ST_{1i}^a + ST_{1i}^b + ST_{1i}^c, \quad \text{say.} \end{aligned} \tag{30}$$

In order to evaluate each term, we use the following lemma.

Lemma A.3 *Under the null hypothesis, as both N and T approach infinity simultaneously, (i) $E\|J_{0i}\|^2 \le C$, $\frac{1}{\sqrt{N}}\sum_{i=1}^N \|J_{0i}\|^2 = O_p(\sqrt{N})$, and $\|\bar{J}_0\| = O_p(\frac{1}{\sqrt{N}})$; (ii) $\|J_1\| = O(1)$; (iii) $E\|J_2\|^2 \le \frac{C}{T}$ and $\|J_2\| = O_p(\frac{1}{\sqrt{T}})$; (iv) $K_{11} = O(1)$, $K_{12} = K_{21}' = O_p(\frac{1}{\sqrt{T}})$, $K_{22} = O_p(1)$, $K^{11} = K_{11}^{-1} + O_p(\frac{1}{T})$, $K_{21} = K_{21}' = O_p(\frac{1}{\sqrt{T}})$, and $K^{22} = O_p(1)$; (v) $E\|J_{3i}\|^2 \le C$, $\frac{1}{\sqrt{N}}\sum_{i=1}^N \|J_{3i}\|^2 = O_p(\sqrt{N})$, and $\|\bar{J}_3\| = O_p(\frac{1}{\sqrt{N}})$; (vi) $E\|J_{4i}\|^2 \le C$, $\frac{1}{\sqrt{N}}\sum_{i=1}^N \|J_{4i}\|^2 = O_p(\sqrt{N})$, and $\bar{J}_4 = O_p(\frac{\sqrt{T}}{N})$; and (vii) $\frac{1}{\sqrt{N}}\sum_{i=1}^N \|J_{0i}\|\|J_{\ell i}\| = O_p(\sqrt{N})$ for $\ell, m = 3, 4$ and $\frac{1}{\sqrt{N}}\sum_{i=1}^N \|J_{3i}\|\|J_{4i}\| = O_p(\sqrt{N})$.*

Since $ST_{1i}^0 = (J_{0i} - J_1K_{11}^{-1}J_{3i})'(J_{0i} - J_1K_{11}^{-1}J_{3i})$, we have using Lemma A.3,

$$\begin{aligned} \left|\frac{1}{\sqrt{N}}\sum_{i=1}^N (ST_{1i}^a - ST_{1i}^0)\right| &= \frac{1}{\sqrt{N}}\sum_{i=1}^N \|J_1(K^{11} - K_{11}^{-1})J_{3i}\|^2 \\ &\quad + \frac{2}{\sqrt{N}}\sum_{i=1}^N \|(J_{0i} - J_1K_{11}^{-1}J_{3i})'\{J_1(K^{11} - K_{11}^{-1})J_{3i}\| \\ &= O_p\left(\frac{\sqrt{N}}{T^2}\right) + O_p\left(\frac{\sqrt{N}}{T}\right), \end{aligned}$$

which converges to 0 in probability when both N and T approach infinity because $N/T \to 0$ by Assumption 1(iii).

In exactly the same manner, we have

$$\frac{1}{\sqrt{N}}\sum_{i=1}^{N}\|ST_{1i}^{b}\| = O_p\left(\frac{\sqrt{N}}{T}\right) \quad \text{and} \quad \frac{1}{\sqrt{N}}\sum_{i=1}^{N}\|ST_{1i}^{c}\| = O_p\left(\frac{\sqrt{N}}{T}\right).$$

Therefore, we obtained (25).

To prove (26) and (27), note that

$$\left|\frac{1}{\sqrt{N}}\sum_{i=1}^{N} ST_{2i}\right| \leq C\frac{\sqrt{N}}{T^2}\bar{\varepsilon}' M_{w^*}L'LM_{w^*}\bar{\varepsilon} \quad \text{and} \quad \left|\frac{1}{\sqrt{N}}\sum_{i=1}^{N} ST_{3i}\right| \leq C\frac{\sqrt{N}}{T^2}\bar{\varepsilon}' M_{w^*}L'LM_{w^*}\bar{\varepsilon}.$$

Then, it is sufficient to show that $\frac{\sqrt{N}}{T^2}\bar{\varepsilon}' M_{w^*}L'LM_{w^*}\bar{\varepsilon} \overset{p\ (N,T)}{\longrightarrow} 0$, which can be proved by noting that $\left\|\frac{1}{T}LM_{w^*}\bar{\varepsilon}\right\| = O_p\left(\frac{1}{\sqrt{N}}\right)$, using expression (29) and Lemma A.3. We thus obtain the result for the augmented KPSS test statistic.

To derive the limiting distribution of the LM test statistic, we first note that under H_0,

$$\begin{aligned}(I_N \otimes M_z)\mathbf{y} &= (\gamma \otimes M_z\mathbf{f}) + (I_N \otimes M_z)\varepsilon \\ &\sim N\left(0,\ A \otimes M_z\right) = (A^{1/2} \otimes M_z)\eta,\end{aligned}$$

where $\eta = [\eta_1', \cdots, \eta_N']' \sim N(0, I_N \otimes I_T)$. Then, $\overline{LM}$ can be expressed as

$$\overline{LM} = \frac{1}{NT^2}\eta'(I_N \otimes M_zL)(A^{-1} \otimes I_T)(I_N \otimes L'M_z)\eta. \tag{31}$$

We first investigate the matrix A. Note that A^{-1} can be expressed as

$$A^{-1} = (\sigma_f^2\gamma\gamma' + I_N)^{-1} = \left(I_N - \frac{1}{1+\sigma_f^2\gamma'\gamma}\sigma_f^2\gamma\gamma'\right).$$

Since $\text{rk}(\gamma\gamma') = 1$ and $(\gamma\gamma')\gamma = (\gamma'\gamma)\gamma$, the $(N-1)$ eigenvalues of $\gamma\gamma'$ are 0 and the non-zero eigenvalue is $\gamma'\gamma$, for which the corresponding eigenvector is γ. Then, there exists an $N \times N$ orthonormal matrix P such that $P'P = PP' = I_N$ and $P'\gamma\gamma'P = \text{diag}\{\gamma'\gamma, 0, \cdots, 0\} \equiv \Lambda_\gamma$. This implies that

$$P'A^{-1}P = I_N - \frac{1}{1+\sigma_f^2\gamma'\gamma}\sigma_f^2\Lambda_\gamma = \text{diag}\left\{\frac{1}{1+\sigma_f^2\gamma'\gamma}, 1, \cdots, 1\right\} \equiv \Lambda_A^{-1}. \tag{32}$$

By inserting (32) into (31), we obtain

$$\begin{aligned}
\overline{LM} &= \frac{1}{NT^2}\eta'(I_N \otimes M_z L)(PP'\sigma_\varepsilon^2 A^{-1}PP' \otimes I_T)(I_N \otimes L'M_z)\eta \\
&= \frac{1}{NT^2}\eta^{*\prime}(I_N \otimes M_z L)(\Lambda_A^{-1} \otimes I_T)(I_N \otimes L'M_z)\eta^* \\
&= \frac{1}{1+\sigma_f^2\gamma'\gamma}\frac{1}{NT^2}\eta_1^{*\prime} M_z LL'M_z \eta_1^* + \frac{1}{NT^2}\sum_{i=2}^{N}\eta_i^{*\prime} M_z LL'M_z\eta_i^*,
\end{aligned}$$

where $\eta^* = [\eta_1^{*\prime}, \cdots, \eta_N^{*\prime}]' = (P \otimes I_T)\eta \sim N(0, I_N \otimes I_T)$. Note that the first term converges to zero in probability as both N and T approach infinity, while the second term has the same structure as ST_{1i}^0. We then obtain the result for the LM test statistic.■

Proof of Theorem 2

We first note that Lemma A.3 still holds under H_1^ℓ using the fact that $\bar{y}_t^* = \bar{r}_t + \bar{\gamma} f_t + \bar{\varepsilon}_t$. Let $J_{0i}^r = T^{-1}L\mathbf{r}_i$, $[J_{3i}^{r\prime}, J_{4i}^{r\prime}]' = D^{-1}W^{*\prime}\mathbf{r}_i = [(D_\tau^{-1}Z'\mathbf{r}_i)', (T^{-1/2}\bar{\mathbf{y}}^{*\prime}\mathbf{r}_i)']'$, $\bar{J}_0^r = N^{-1}\sum_{i=1}^N J_{0i}^r$, $\bar{J}_3^r = N^{-1}\sum_{i=1}^N J_{3i}^r$, and $\bar{J}_4^r = N^{-1}\sum_{i=1}^N J_{4i}^r$. Under H_1^ℓ, we have the following lemma.

Lemma A.4 *Under the local alternative H_1^ℓ, as both N and T approach infinity simultaneously, (i) $E\|J_{0i}^r\|^2 \leq \frac{C}{\sqrt{N}}$, $\frac{1}{\sqrt{N}}\sum_{i=1}^N \|J_{0i}^r\|^2 = O_p(1)$, and $\|\bar{J}_0^r\| = O_p(\frac{1}{N^{3/4}})$; (ii) $E\|J_{3i}^r\|^2 \leq \frac{C}{\sqrt{N}}$, $\frac{1}{\sqrt{N}}\sum_{i=1}^N \|J_{3i}^r\|^2 = O_p(1)$, and $\|\bar{J}_3^r\| = O_p(\frac{1}{N^{3/4}})$; (iii) $E\|J_{4i}^r\|^2 \leq \frac{C}{\sqrt{NT}}$, $\frac{1}{\sqrt{N}}\sum_{i=1}^N \|J_{4i}^r\|^2 = O_p(\frac{1}{T})$, and $\|\bar{J}_4^r\| = O_p(\frac{1}{N^{3/4}\sqrt{T}})$; and (iv) $\frac{1}{\sqrt{N}}\sum_{i=1}^N \|J_{0i}^r\|\|J_{3i}^r\| = O_p(1)$, $\frac{1}{\sqrt{N}}\sum_{i=1}^N \|J_{0i}^r\|\|J_{4i}^r\| = O_p(\frac{1}{\sqrt{T}})$, and $\frac{1}{\sqrt{N}}\sum_{i=1}^N \|J_{3i}^r\|\|J_{4i}^r\| = O_p(\frac{1}{\sqrt{T}})$.*

Note that, under the local alternative,

$$\frac{1}{T}LM_{w^*}\mathbf{y}_i = \frac{1}{T}LM_{w^*}\varepsilon_i - \frac{\tilde{\gamma}_i}{T}LM_{w^*}\bar{\varepsilon} + \frac{1}{T}LM_{w^*}\mathbf{r}_i - \frac{\tilde{\gamma}_i}{T}LM_{w^*}\bar{\mathbf{r}}.$$

Using Lemmas A.3 and A.4, it can be shown that

$$\left\|\frac{1}{T}LM_{w^*}\bar{\varepsilon}\right\| = O_p\left(\frac{1}{\sqrt{N}}\right) \quad \text{and} \quad \left\|\frac{1}{T}LM_{w^*}\bar{\mathbf{r}}\right\| = O_p\left(\frac{1}{N^{3/4}}\right),$$

and thus, since $|\tilde{\gamma}_i| \leq C$, we can see that both

$$\left\|\frac{1}{\sqrt{N}T^2}\sum_{i=1}^N \tilde{\gamma}_i\varepsilon_i' M_{w^*}L'LM_{2^*}\bar{\mathbf{r}}\right\| \quad \text{and} \quad \left\|\frac{1}{\sqrt{N}T^2}\sum_{i=1}^N \tilde{\gamma}_i\mathbf{r}_i' M_{w^*}L'LM_{2^*}\bar{\varepsilon}\right\|$$

are $O_p(N^{-3/4})$. Therefore, the cross products between the terms related with ε_i and $\bar{\mathbf{r}}$, $\mathbf{r}_i$ and $\bar{\varepsilon}$, and $\bar{\varepsilon}$ and $\bar{\mathbf{r}}$ converge to zero in probability as both N and T approach infinity.

In addition, using expression (28) and Lemmas A.3 and A.4, it is observed that both

$$\|\frac{1}{T}LM_{w^*}\varepsilon_i - (J_{0i} - J_1 K_{11}^{-1} J_{3i})\| \quad \text{and} \quad \|\frac{1}{T}LM_{w^*}\mathbf{r}_i - (J_{0i}^r - J_1 K_{11}^{-1} J_{3i}^r)\|$$

are $O_p(T^{-1/2})$, which implies that we only have to consider $(J_{0i} - J_1 K_{11}^{-1} J_{3i})$ and $(J_{0i}^r - J_1 K_{11}^{-1} J_{3i}^r)$ in the limit. Moreover, the cross product between these two terms can be seen to be negligible. Therefore, we have

$$\begin{aligned} Z_A &= \frac{1}{\zeta\sqrt{N}} \sum_{i=1}^{N} \left\{ \left(J_{0i} - J_1 K_{11}^{-1} J_{3i}\right)' \left(J_{0i} - J_1 K_{11}^{-1} J_{3i}\right) - \xi \right\} \\ &\quad + \frac{1}{\zeta N} \sum_{i=1}^{N} \sqrt{N} \left(J_{0i}^r - J_1 K_{11}^{-1} J_{3i}^r\right)' \left(J_{0i} - J_1 K_{11}^{-1} J_{3i}^r\right) + o_p(1). \end{aligned}$$

The first term weakly converges to a standard normal distribution as proved in Theorem 1, whereas the probability limit of the second term is obtained by applying Lemma A.2 (i). To see this, we first note that, using Lemma A.1,

$$E\left[\sqrt{N} \left(J_{0i}^r - J_1 K_{11}^{-1} J_{3i}^r\right)' \left(J_{0i}^r - J_1 K_{11}^{-1} J_{3i}^r\right)\right] = \frac{11}{12600} c^2 + O\left(\frac{1}{T^2}\right),$$

while the second moment is bounded above uniformly over N and T using (23). On the other hand, since $N^{1/4} T r_{[Tr]} \overset{T}{\Longrightarrow} c B_i^v(r)$, we can see that

$$\sqrt{N} \left(J_{0i}^r - J_1 K_{11}^{-1} J_{3i}^r\right)' \left(J_{0i} - J_1 K_{11}^{-1} J_{3i}^r\right) \overset{T}{\Longrightarrow} c^2 \int_0^1 F_i^v(r)^2 dr,$$

whose moment is $11c^2/12600$ by direct calculation. Then, by Lemma A.2 (i), we have

$$\frac{1}{\zeta\sqrt{N}} \sum_{i=1}^{N} \left(J_{0i}^r - J_1 K_{11}^{-1} J_{3i}^r\right)' \left(J_{0i}^r - J_1 K_{11}^{-1} J_{3i}^r\right) \overset{p\ (N,T)}{\longrightarrow} \frac{c^2}{\zeta} E\left[\int_0^1 F_i^v(r)^2 dr\right] = \frac{11}{12600} \frac{c^2}{\zeta}.$$

When $z_t = 1$, the above probability limit can be shown to be $c^2/(90\zeta)$ in exactly the same manner.

In order to derive the limiting distribution of the LM test statistic, note that $(I_N \otimes M_z)\mathbf{y} = (I_N \otimes M_z)\mathbf{r} + (A^{1/2} \otimes M_z)\eta$. Then, the denominator of the LM test statistic can be expressed as

$$\sqrt{N}(\overline{LM}-\xi) = \frac{1}{\sqrt{N}T^2}\left\{\eta'(A^{-2}\otimes M_zLL'M_z)\eta-\xi\right\} + \frac{1}{\sqrt{N}T^2}\mathbf{r}'(A^{-2}\otimes M_zLL'M_z)\mathbf{r}+\frac{2}{\sqrt{N}T^2}\eta'(A^{-2}\otimes M_zLL'M_z)\varepsilon. \tag{33}$$

The first term on the right-hand side of (33) converges in distribution to a standard normal distribution as proved in Theorem 1.

Since $A^{-2}=P\Lambda_A^{-2}P'$, the second term on the right-hand side of (33) is expressed as

$$\frac{1}{\sqrt{N}T^2}\mathbf{r}'(A^{-2}\otimes M_zLL'M_z)\mathbf{r}=\frac{1}{\sqrt{N}T^2}\mathbf{r}'(P\otimes I_T)(\Lambda_A^{-2}\otimes M_zLL'M_z)(P\otimes I_T)\mathbf{r}.$$

Note that $(P\otimes I_T)\mathbf{r}=(P\otimes I_T)(I_N\otimes L)\mathbf{v}=(I_N\otimes L)\mathbf{v}^*=\mathbf{r}^*$, where $\mathbf{v}^*\equiv(P\otimes I_T)\mathbf{v}\sim N(0,\rho(I_N\otimes I_T))$ and $\mathbf{r}^*\equiv(I_N\otimes L)\mathbf{v}^*$. Using this expression, we have

$$\begin{aligned}\frac{1}{\sqrt{N}T^2}\mathbf{r}'(A^{-2}\otimes M_zLL'M_z)\mathbf{r} &= \frac{1}{\sqrt{N}T^2}\mathbf{r}^{*\prime}(\Lambda_A^{-2}\otimes M_zLL'M_z)\mathbf{r}^*\\ &= \frac{1}{(1+\sigma_f^2\gamma'\gamma)}\frac{1}{\sqrt{N}T^2}\mathbf{r}_1^{*\prime}M_zLL'M_z\mathbf{r}_1^*+\frac{1}{\sqrt{N}T^2}\sum_{i=2}^{N}\mathbf{r}_i^{*\prime}M_zLL'M_z\mathbf{r}_i^*.\end{aligned}$$

It is not difficult to see that the first term on the right-hand side converges to zero as both N and T approach infinity, while

$$\frac{1}{\sqrt{N}T^2}\sum_{i=2}^{N}\mathbf{r}_i^{*\prime}M_zLL'M_z\mathbf{r}_i^*=\frac{1}{\sqrt{N}}\sum_{i=2}^{N}\left(J_{0i}^{r*}-J_1K_{11}^{-1}J_{3i}^{r*}\right)'\left(J_{0i}^{r*}-J_1K_{11}^{-1}J_{3i}^{r*}\right), \tag{34}$$

where J_{0i}^{r*} and J_{3i}^{r*} are defined in the same way as J_{0i}^{r} and J_{3i}^{r} with r_t being replaced by r_t^*. Since $\mathbf{r}^*$ has the same distribution as $\mathbf{r}$, (34) converges in probability to $11c^2/12600$ as proved for the case of Z_A.

Similarly, we can see that the third term on the right-hand side of (33) converges to zero in probability as proved for the case of Z_A.■

Proof of Theorem 3

Lemma A.5 *Under the fixed alternative H_1, as both N and T approach infinity simultaneously, (i) $E\|J_{0i}\|^2\le C$, $\frac{1}{\sqrt{N}}\sum_{i=1}^N\|J_{0i}\|^2=O_p(\sqrt{N})$, and $\|\bar{J}_0\|=O_p(\frac{1}{\sqrt{N}})$; (ii)*

$\|J_1\| = O(1)$; *(iii)* $E\|J_2\|^2 \leq C\frac{T}{N}$ *and* $\|J_2\| = O_p(\frac{\sqrt{T}}{\sqrt{N}})$; *(iv)* $K_{11} = O(1)$, $K_{12} = K_{21}' = O_p(\frac{\sqrt{T}}{\sqrt{N}})$, *and* $K_{22} = O_p(\frac{T}{N})$; *(v)* $E\|J_{3i}\|^2 \leq C$, $\frac{1}{\sqrt{N}}\sum_{i=1}^{N}\|J_{3i}\|^2 = O_p(\sqrt{N})$, *and* $\|\bar{J}_3\| = O_p(\frac{1}{\sqrt{N}})$; *(vi)* $E\|J_{4i}\|^2 \leq C\frac{T}{N}$, $\frac{1}{\sqrt{N}}\sum_{i=1}^{N}\|J_{4i}\|^2 = O_p(\frac{T}{\sqrt{N}})$, *and* $\bar{J}_4 = O_p(\frac{\sqrt{T}}{N})$; *and (vii)* $\frac{1}{\sqrt{N}}\sum_{i=1}^{N}\|J_{0i}\|\|J_{3i}\| = O_p(\sqrt{N})$, $\frac{1}{\sqrt{N}}\sum_{i=1}^{N}\|J_{0i}\|\|J_{4i}\| = O_p(\sqrt{T})$, *and* $\frac{1}{\sqrt{N}}\sum_{i=1}^{N}\|J_{3i}\|\|J_{4i}\| = O_p(\sqrt{T})$.

Lemma A.6 *Under the fixed alternative H_1, as both N and T approach infinity simultaneously, (i)* $E\|J_{0i}^r\|^2 \leq CT^2$, $\frac{1}{\sqrt{N}}\sum_{i=1}^{N}\|J_{0i}^r\|^2 = O_p(\sqrt{N}T^2)$, *and* $\|\bar{J}_0^r\| = O_p(\frac{T}{\sqrt{N}})$; *(ii)* $E\|J_{3i}^r\|^2 \leq CT^2$, $\frac{1}{\sqrt{N}}\sum_{i=1}^{N}\|J_{3i}^r\|^2 = O_p(\sqrt{N}T^2)$, *and* $\|\bar{J}_3^r\| = O_p(\frac{T}{\sqrt{N}})$; *(iii)* $E\|J_{4i}^r\|^2 \leq C\frac{T^3}{N}$, $\frac{1}{\sqrt{N}}\sum_{i=1}^{N}\|J_{4i}^r\|^2 = O_p(\sqrt{N}T^3)$, *and* $\|\bar{J}_4^r\| = O_p(\frac{T\sqrt{T}}{N})$; *and (iv)* $\frac{1}{\sqrt{N}}\sum_{i=1}^{N}\|J_{0i}^r\|\|J_{3i}^r\| = O_p(\sqrt{N}T^2)$, $\frac{1}{\sqrt{N}}\sum_{i=1}^{N}\|J_{0i}^r\|\|J_{4i}^r\| = O_p(T^2\sqrt{T})$ *and* $\frac{1}{\sqrt{N}}\sum_{i=1}^{N}\|J_{3i}^r\|\|J_{4i}^r\| = O_p(T^2\sqrt{T})$.

Using Lemmas A.5 and A.6, it can be shown that

$$\left\|\frac{1}{T^2}LM_{w^*}\varepsilon_i\right\| = O_p\left(\frac{1}{T}\right), \quad \left\|\frac{1}{T^2}LM_{w^*}\bar{\varepsilon}\right\| = O_p\left(\frac{1}{\sqrt{NT}}\right), \quad \left\|\frac{1}{T^2}LM_{w^*}\bar{\mathbf{r}}\right\| = O_p\left(\frac{1}{\sqrt{N}}\right),$$

which implies that

$$\frac{1}{\sqrt{N}T^2}Z_A = \frac{1}{\zeta}\frac{1}{N}\sum_{i=1}^{N}\frac{1}{T^4}\mathbf{r}_i' M_{w^*}L'LM_{w^*}\mathbf{r}_i + o_p(1).$$

We decompose the first term on the right-hand side such that

$$\frac{1}{N}\sum_{i=1}^{N}\frac{1}{T^2}LM_{w^*}\mathbf{r}_i = \frac{1}{T}J_{0i}^r - \left[J_1, \frac{\sqrt{N}}{\sqrt{T}}J_2\right]\left[\begin{array}{cc} K_{11} & \frac{\sqrt{N}}{\sqrt{T}}K_{12} \\ \frac{\sqrt{N}}{\sqrt{T}}K_{21} & \frac{N}{T}K_{22}\end{array}\right]^{-1}\left[\begin{array}{c}\frac{1}{T}J_{3i}^r \\ \frac{\sqrt{N}}{T\sqrt{T}}J_{4i}^r\end{array}\right].$$

Using this expression and letting $\underline{K} = D_2^{-1}KD_2^{-1}$, where $D_2 = diag\{1, \sqrt{T}/\sqrt{N}\}$, we have

$$\begin{aligned}
&\frac{1}{N}\sum_{i=1}^{N}\frac{1}{T^4}\mathbf{r}_i M_{w^*}L'LM_{w^*}\mathbf{r}_i \\
&= \frac{1}{N}\sum_{i=1}^{N}\frac{1}{T^2}J_{0i}^{r\prime}J_{0i}^r - 2tr\left(\underline{K}^{-1}\frac{1}{N}\sum_{i=1}^{N}\left[\begin{array}{c}\frac{1}{T}J_{3i}^r \\ \frac{\sqrt{N}}{T\sqrt{T}}J_{4i}^r\end{array}\right]\left[\frac{1}{T}J_{0i}^{r\prime}J_1, \frac{\sqrt{N}}{T\sqrt{T}}J_{0i}^{r\prime}J_2\right]\right) \qquad (35) \\
&\quad +tr\left(\underline{K}^{-1}\left[\begin{array}{cc} J_1'J_1 & \frac{\sqrt{N}}{\sqrt{T}}J_1'J_2 \\ \frac{\sqrt{N}}{\sqrt{T}}J_2'J_1 & \frac{N}{T}J_2'J_2\end{array}\right]\underline{K}^{-1}\frac{1}{N}\sum_{i=1}^{N}\left[\begin{array}{c}\frac{1}{T}J_{3i}^r \\ \frac{\sqrt{N}}{T\sqrt{T}}J_{4i}^r\end{array}\right]\left[\begin{array}{cc}\frac{1}{T}J_{3i}^{r\prime} & \frac{\sqrt{N}}{T\sqrt{T}}J_{4i}^{r\prime}\end{array}\right]\right).
\end{aligned}$$

By applying Lemmas A.1 and A.2, it can be shown that the joint limits of the three terms on the right-hand side of (35) are the same as the sequential limits, which are given by

$$\frac{1}{N}\sum_{i=1}^{N}\frac{1}{T^2}J_{0i}^{r\prime}J_{0i}^{r}\overset{p\ (N,T)}{\longrightarrow}\sigma_v^2 E_{vi}\left[\int_0^1\left(\int_0^r B_i^v(s)ds\right)^2 dr\right], \tag{36}$$

$$\begin{aligned} &2tr\left(\underline{K}^{-1}\frac{1}{N}\sum_{i=1}^{N}\begin{bmatrix}\frac{1}{T}J_{3i}^{r}\\ \frac{\sqrt{N}}{T\sqrt{T}}J_{4i}^{r}\end{bmatrix}\begin{bmatrix}\frac{1}{T}J_{0i}^{r\prime}J_1, \frac{\sqrt{N}}{T\sqrt{T}}J_{0i}^{r\prime}J_2\end{bmatrix}\right) \qquad (37)\\ &\overset{(N,T)}{\Longrightarrow} 2\sigma_v^2E_{vi}\left[\left(\int_0^1 z_2(t)'B_i^v(t)dt\right)\left(\int_0^1 z_2(t)z_2(t)'dt\right)^{-1}\int_0^1\int_0^r z_s(s)ds\int_0^r B_i^v(t)dtdr\right], \end{aligned}$$

$$\begin{aligned} &tr\left(\underline{K}^{-1}\begin{bmatrix}J_1'J_1 & \frac{\sqrt{N}}{\sqrt{T}}J_1'J_2\\ \frac{\sqrt{N}}{\sqrt{T}}J_2'J_1 & \frac{N}{T}J_2'J_2\end{bmatrix}\underline{K}^{-1}\frac{1}{N}\sum_{i=1}^{N}\begin{bmatrix}\frac{1}{T}J_{3i}^{r}\\ \frac{\sqrt{N}}{T\sqrt{T}}J_{4i}^{r}\end{bmatrix}\begin{bmatrix}\frac{1}{T}J_{3i}^{r\prime} & \frac{\sqrt{N}}{T\sqrt{T}}J_{4i}^{r\prime}\end{bmatrix}\right)\\ &\overset{(N,T)}{\Longrightarrow}\ \sigma_v^2E_{vi}\left[\int_0^1 z_2(t)'B_i^v(t)dt\left(\int_0^1 z_2(t)z_2(t)'dt\right)^{-1}\right.\\ &\qquad\left.\times\int_0^1\int_0^r z_2(s)ds\int_0^r z_2(s)'dsdr\left(\int_0^1 z_2(t)z_2(t)'dt\right)^{-1}\int_0^1 z_2(t)'B_i^v(t)dt\right]. \qquad (38) \end{aligned}$$

Using these results, we obtain the joint weak limit of Z_A under H_1 because $\sigma_v^2=\sigma_\varepsilon^2\rho=\rho$.

For the LM test statistic, we can see using expression (33) and Lemmas A.5 and A.6 that

$$\begin{aligned} \frac{1}{\sqrt{N}T^2}Z_{LM} &= \frac{1}{\zeta NT^4}\mathbf{r}'\left(A^{-2}\otimes M_zLL'M_z\right)\mathbf{r}+o_p(1)\\ &= \frac{1}{\zeta NT^2}\sum_{i=2}^{N}\left(J_{0i}^{r*}-J_1K_{11}^{-1}J_{3i}^{r*}\right)'\left(J_{0i}^{r*}-J_1K_{11}^{-1}J_{3i}^{r*}\right)+o_p(1)\\ &\overset{p\ (N,T)}{\longrightarrow} \frac{\sigma_v^2}{\zeta}E\left[\int_0^1 F_i^{v*}(r)^2dr\right]. \end{aligned}$$

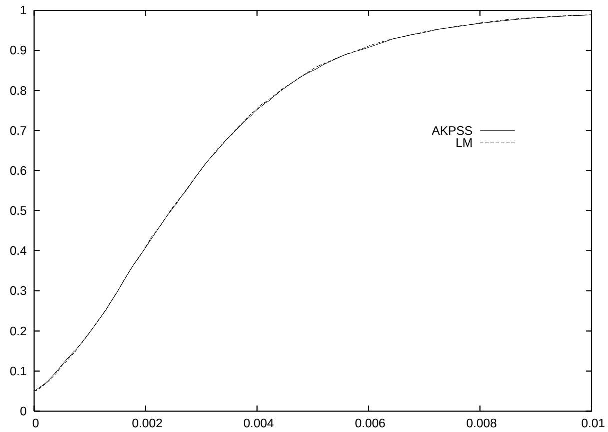
Because $F_i^{v*}(r)$ has the same distribution as $F_i^v(r)$, we obtain the theorem.■

Table 1. Size of the tests

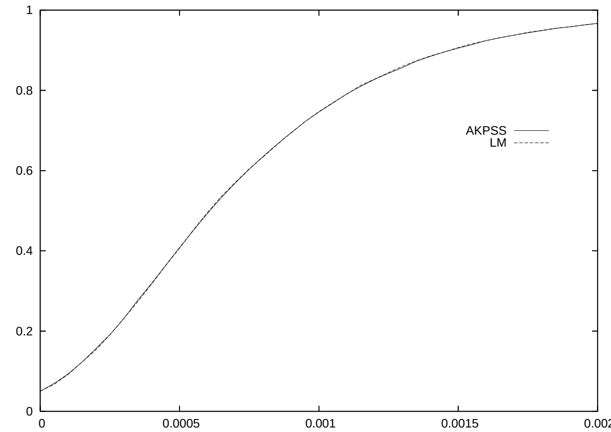
		constant case				trend case			
		SCC		WCC		SCC		WCC	
N	T	Z_A	Z_{LM}	Z_A	Z_{LM}	Z_A	Z_{LM}	Z_A	Z_{LM}
10	50	0.049	0.033	0.026	0.061	0.040	0.022	0.018	0.062
	100	0.053	0.034	0.033	0.067	0.045	0.026	0.024	0.063
	200	0.057	0.036	0.036	0.066	0.048	0.026	0.026	0.063
20	50	0.052	0.041	0.032	0.065	0.040	0.029	0.023	0.060
	100	0.057	0.042	0.040	0.067	0.043	0.030	0.023	0.058
	200	0.058	0.045	0.041	0.067	0.047	0.031	0.029	0.060
30	50	0.053	0.041	0.034	0.060	0.041	0.031	0.021	0.056
	100	0.056	0.043	0.037	0.060	0.047	0.032	0.027	0.057
	200	0.054	0.040	0.037	0.059	0.046	0.032	0.029	0.057
50	50	0.051	0.046	0.034	0.060	0.041	0.036	0.022	0.058
	100	0.055	0.044	0.036	0.058	0.049	0.039	0.032	0.061
	200	0.056	0.046	0.042	0.063	0.048	0.036	0.033	0.055
100	50	0.061	0.047	0.030	0.058	0.046	0.038	0.019	0.052
	100	0.064	0.046	0.036	0.056	0.060	0.040	0.029	0.056
	200	0.060	0.040	0.038	0.052	0.064	0.040	0.033	0.055

Table 2. Power of the tests

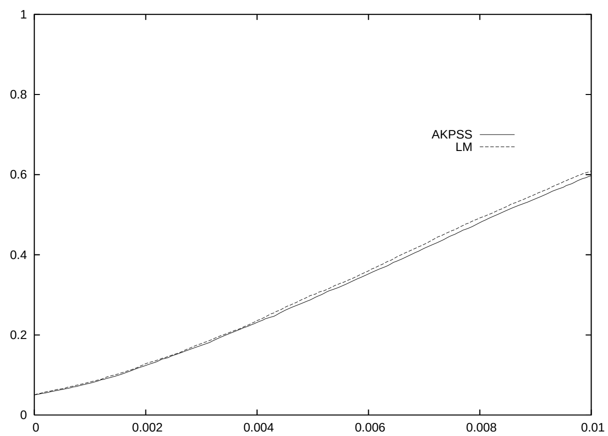
N	T	constant case SCC Z_A	constant case SCC Z_{LM}	constant case WCC Z_A	constant case WCC Z_{LM}	trend case SCC Z_A	trend case SCC Z_{LM}	trend case WCC Z_A	trend case WCC Z_{LM}
10	50	0.055	0.038	0.034	0.069	0.045	0.025	0.019	0.066
		0.145	0.109	0.092	0.181	0.061	0.037	0.030	0.088
		0.882	0.862	0.796	0.923	0.356	0.297	0.243	0.453
	100	0.087	0.059	0.055	0.106	0.049	0.028	0.025	0.070
		0.520	0.462	0.434	0.588	0.143	0.097	0.086	0.193
		1.000	1.000	0.997	1.000	0.948	0.929	0.904	0.973
	200	0.224	0.178	0.171	0.270	0.073	0.044	0.043	0.103
		0.970	0.962	0.955	0.985	0.582	0.506	0.488	0.670
		1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
20	50	0.062	0.049	0.039	0.077	0.042	0.032	0.025	0.063
		0.202	0.178	0.148	0.238	0.066	0.053	0.039	0.096
		0.989	0.988	0.963	0.994	0.559	0.526	0.447	0.646
	100	0.102	0.081	0.072	0.118	0.053	0.037	0.032	0.069
		0.759	0.729	0.695	0.800	0.204	0.164	0.145	0.255
		1.000	1.000	1.000	1.000	0.999	0.999	0.996	1.000
	200	0.341	0.303	0.291	0.388	0.097	0.071	0.063	0.122
		1.000	0.999	0.999	1.000	0.834	0.801	0.781	0.876
		1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
30	50	0.067	0.054	0.041	0.076	0.041	0.033	0.021	0.057
		0.254	0.226	0.181	0.287	0.071	0.059	0.039	0.099
		0.999	0.999	0.988	0.999	0.713	0.692	0.589	0.779
	100	0.126	0.099	0.088	0.135	0.062	0.045	0.038	0.075
		0.888	0.870	0.841	0.908	0.276	0.231	0.201	0.321
		1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	200	0.434	0.392	0.376	0.462	0.122	0.087	0.079	0.140
		1.000	1.000	1.000	1.000	0.943	0.926	0.912	0.957
		1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
50	50	0.066	0.056	0.044	0.075	0.044	0.041	0.026	0.063
		0.342	0.318	0.253	0.373	0.084	0.081	0.051	0.114
		1.000	1.000	0.998	1.000	0.878	0.877	0.787	0.913
	100	0.141	0.121	0.105	0.151	0.066	0.054	0.044	0.081
		0.975	0.971	0.958	0.980	0.373	0.343	0.292	0.419
		1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	200	0.582	0.554	0.530	0.609	0.143	0.118	0.108	0.164
		1.000	1.000	1.000	1.000	0.992	0.991	0.989	0.994
		1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
100	50	0.085	0.066	0.044	0.080	0.054	0.042	0.023	0.057
		0.539	0.499	0.391	0.541	0.124	0.108	0.061	0.138
		1.000	1.000	1.000	1.000	0.991	0.991	0.960	0.994
	100	0.218	0.174	0.146	0.200	0.090	0.066	0.050	0.085
		1.000	1.000	0.999	1.000	0.629	0.573	0.501	0.626
		1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	200	0.832	0.803	0.780	0.827	0.237	0.179	0.158	0.223
		1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
		1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000



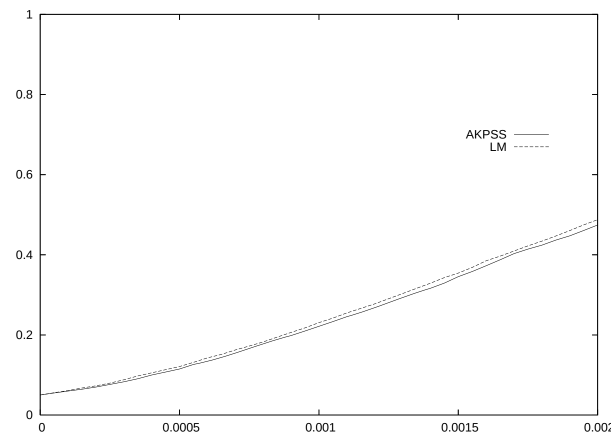
(i-a) $N = 50$, $T = 50$, constant, SCC



(i-b) $N = 50$, $T = 100$, constant, SCC



(i-c) $N = 50$, $T = 50$, trend, SCC



(i-d) $N = 50$, $T = 100$, trend, SCC

Figure 1: Finite sample power under restrictive assumptions