QUEEN'S UNIVERSITY BELFAST | SCHOOL OF MATHEMATICS AND PHYSICS

Modules offered on Mathematics programmes in 2024-25

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Level 1 Modules

- **MTH1011 Introduction to Algebra and Analysis (Full year)**

Pre-requisite: A Level Mathematics Grade A

Lecturers: Dr Y-F Lin, Dr T Huettemann, Dr A Blanco

Introduction

The module covers fundamental aspects of basic mathematics, in particular the foundations (logic, proof techniques, concepts of set theory, number systems), introduction to linear algebra (systems of linear equations, vector spaces, bases, dimension) and introduction to analysis (limits, continuity, derivatives). The module builds a solid foundation for further mathematical studies.

Content

Elementary logic and set theory, number systems (including integers, rationals, reals and complex numbers), bounds, suprema and infima, basic combinatorics, functions.

Sequences of real numbers, the notion of convergence of a sequence, completeness, the Bolzano-Weierstrass theorem, limits of series of non-negative reals and convergence tests.

Analytical definition of continuity, limits of functions and derivatives in terms of a limit of a function. Properties of continuous and differentiable functions. L'Hopital's rule, Rolle's theorem, mean-value theorem.

Matrices and systems of simultaneous linear equations, vector spaces, linear dependence, basis, dimension.

Assessment

Homework submission 10%
Class tests 15% (× 2)
Final Exam 60%

- **MTH1015 Mathematical Reasoning (Sem 1)**

Pre-requisite: A Level Mathematics Grade A

Lecturer: Dr T Todorov

Introduction

Mathematics is not all about numbers: it is also about convincing other people that the given answer is the correct one. This is done by providing verifiable arguments. The word “verifiable” is key here: this involves both correct logical reasoning and a way of communicating that makes it easy for others to follow the train of thought.

This module will familiarise the student with aspects of formal logic and common mathematical reasoning techniques (known as “proof by induction” and “proof by contradiction”). On this basis, mathematical communication skills will be developed and applied to various group-working tasks in the tutorials, a written project, and an oral presentation. The skills developed here are fundamental and are used in all areas of mathematics.

This module supplements the two core modules, MTH1011 and MTH1021.

Content

The notion of mathematical statements and elementary logic. Mathematical symbols and notation. The language of sets. The concept of mathematical proof, and typical examples. Communicating mathematics to others. Use of AI for problem solving and drafting text: challenges and applications.

Assessment

Assignments 40%
Oral Presentations 25%
Report 25%
Tutorial Contribution 10%

- **MTH1021 Mathematical Methods 1 (Full year)**

Pre-requisite: A Level Mathematics Grade A

Lecturers: Dr D Green, Dr G De Chiara, Dr C Ramsbottom

Introduction

The module begins with a revision of basic calculus and elementary functions. These fundamental topics form the basis for investigating the properties of functions using derivatives, integrals and power series. Differential equations, complex numbers, vector analysis, matrix manipulation and calculus in several variables are covered in the module. It thus provides many useful mathematical tools for applications in mechanics, statistics, quantum theory etc. Together with the module MTH1011, these methods build a solid foundation for further mathematical studies at higher levels.

Content

Review of A-level calculus: elementary functions and their graphs, domains and ranges, trigonometric functions, derivatives and differentials, integration. Maclaurin expansion. Complex numbers and Euler's formula.

Differential equations (DE); first-order DE: variable separable, linear; second-order linear DE with constant coefficients: homogeneous and inhomogeneous.

Vectors in 3D, definitions and notation, operations on vectors, scalar and vector products, triple products, 2x2 and 3x3 determinants, applications to geometry, equations of a plane and straight line. Rotations and linear transformations in 2D, 2x2 and 3x3 matrices, eigenvectors and eigenvalues.

Newtonian mechanics: kinematics, plane polar coordinates, projectile motion, Newton's laws, momentum, types of forces, simple pendulum, oscillations (harmonic, forced, damped), planetary motion (universal law of gravity, angular momentum, conic sections, Kepler's problem).

Curves in 3D (length, curvature, torsion). Functions of several variables, derivatives in 2D and 3D, Taylor expansion, total differential, gradient (nabla operator), stationary points for a function of two variables. Vector functions; div, grad and curl operators and vector operator identities. Line integrals, double integrals, Green's theorem. Surfaces (parametric form, 2nd-degree surfaces). Curvilinear coordinates, spherical and cylindrical coordinates, orthogonal curvilinear coordinates, Lamé coefficients. Volume and surface integrals, Gauss's theorem, Stokes's theorem. Operators div, grad, curl and Laplacian in orthogonal curvilinear coordinates.

Assessment
Exam 60%
Class Test 15%
Assignment 15%
Homework submission 10%

- **MTH1025 Algorithmic Thinking (Sem 2)**

Pre-requisite: A Level Mathematics Grade A

Lecturer: Dr A Brown

Introduction

Modern mathematics makes extensive use of computers to solve problems. While computers are immensely powerful tools, they require clear, step-by-step instructions to perform even simple tasks. In this module you will learn how to break mathematical operations down into these step-by-step instructions (algorithms), and how to implement those operations in computer code. The skills learned in this module are not only key for work you will do later in your degree but are also highly prized by employers.

Content

Programming fundamentals (variables, operations, logic, loops); using pseudocode to construct algorithms and plan programs; basic programming skills with Python; using software (LaTeX) to present mathematical content, and to solve mathematical problems (python packages: numpy, matplotlib and sympy).

Assessment

Introduction to Python (online course) 20%
Short projects using selected mathematical software 60%
Canvas quizzes 20%

- **SOR1020 Introduction to Probability & Statistics (Full year)**

Pre-requisite: This module is intended for students at stage 1 of either an MMath/MSci or a BSc Mathematics pathway, and a mathematical knowledge and ability commensurate with this stage is assumed.

Lecturers: Dr K Cairns, Dr Z Lin

Introduction

This module introduces students to the fundamentals of probability and statistics. We start at the very beginning with basic probability before introducing both discrete and continuous random variables, and their properties. Typical standard discrete and continuous distributions such as Binomial, Exponential, Poisson and Normal are discussed with particular emphasis on how these distributions relate to systems in real life. This theory is used to provide a foundation for the formulation of statistical models and to introduce methods of parameter estimation. With this knowledge in particular it is possible to carry out hypothesis tests, both parametric and non-parametric, which are frequently used tools to analyse data in real world scenarios, including in important fields such as epidemiology, finance, pharma and in engineering.

Content

Probability: Definitions and laws of probability. Interpretation of probabilities and relationships between probability and statistics. Conditional probability, in particular Bayes Theorem.

Discrete and Continuous Random Variables and Probability Distributions: Key definitions and properties of discrete and continuous random variables and probability distributions. Expected values of discrete and continuous random variables, including properties of expectation and variance operators.

Standard Discrete Distributions: Bernoulli, Geometric, Binomial, Negative Binomial, Hypergeometric and Poisson distributions.

Standard Continuous Random Variables: Uniform, Exponential and Normal distributions to include use of statistical tables, linear combinations of independent normal random variables, central limit theorems and approximations of Binomial and Poisson distributions.

Bivariate Distributions: Key definitions and properties of discrete and continuous bivariate distributions. Properties of independence and expected values; mean, variance, covariance. Correlations coefficients. Means, variances and covariance of linear combinations of random variables.

Statistical Models: Description of "mathematical" modelling. Statistical models. Measurement models, experimental, systematic and random errors; precision and accuracy.

Sampling: Sample surveys. Methods of sampling and probability sampling schemes. Errors in sample surveys. Advantages of sampling. Sampling from infinite populations.

Estimation: Key definitions and properties of estimation. Desirable properties for an estimator and estimation of mean and variance from a single sample and from several samples. Properties of point estimators leading to the method of moments, method of maximum likelihood estimation and method of least squares estimation.

Introduction to Hypothesis Testing: General principles, null and alternative hypotheses, one and two-sided tests, test statistics, critical regions, P-values, significance level, type I and type II errors and power function. Interpretation of results of a significance test, including confidence intervals.

Hypothesis Tests: Parametric tests based upon Normal distribution, t-distribution, F-distribution and chi-squared distribution. Nonparametric tests including the Wilcoxon signed-rank test and Mann-Whitney test.

Statistical Quality Control: Process control for systems including Shewhart control charts, upper and lower control limits, upper and lower warning limits. Analysis of patterns and construction of a control chart for attributes and variables.

Assessment

Class tests 30%
Homework submission 10%
Exam 60%

- **SOR1021 Introduction to Statistical and Operational Research Methods (Sem 2)**

Pre-requisite: This module is intended for students at stage 1 of either an MMath/MSci or a BSc Mathematics pathway, and a mathematical knowledge and ability commensurate with this stage is assumed.

Lecturer: Dr Z Lin

Introduction

Introduction to statistical software for applying the following topics in Operational Research and Statistical Methods:

Content

Introduction to statistical software for applying the following topics in Operational Research and Statistical Methods:

Linear Programming: Characteristics of linear programming models, general form. Graphical solution. Simplex method: standard form of linear programming problem, conversion procedures, basic feasible solutions. Simplex algorithm: use of artificial variables.

Decision Theory: Characteristics of a decision problem. Decision making under uncertainty: maximax, maximin, generalised maximin (Hurwicz), minimax regret criteria. Decision making under risk: Bayes criterion, value of perfect information. Decision tree; Bayesian decision analysis.

Random Sampling and Simulation: Random sample from a finite population, from a probability distribution. Use of random number tables. General method for drawing a random sample from a discrete distribution. Drawing a random sample from a continuous distribution: inverse transformation method, exponential distribution. Dynamic simulation techniques: application to queueing problems. Computer aspects: random number generators, sampling from normal distributions.

Initial Data Analysis: Scales of measurement. Discrete and continuous variables. Sample mean, variance, standard deviation, percentile for ungrouped data; boxplot. Frequency table for grouped discrete data: relative frequency, cumulative frequency, bar diagram; sample mean, variance, percentile. Frequency table for grouped continuous data: stem-and-leaf plot, histogram, cumulative percentage frequency plot; sample mean, variance, percentile. Linear transformation. Bivariate data; scatter diagram, sample correlation coefficient.

Assessment

Timed Exam on Computer 45% (× 2)
Lab engagement 10%

Level 2 Modules

- **MTH2010 Employability for Mathematics (Sem 1)**

Pre-requisites: None

Lecturer: Dr S Moutari

Introduction

This is a 0 CAT point module that is compulsory for students planning to take a placement year. The module consists of 6 lectures and 2 workshops and is assessed 100% by attendance.

Content

Introduction to placement for mathematics and physics students, CV building, international options, interview skills, assessment centres, placement approval, health and safety and wellbeing. Workshops on CV building and interview skills. The module is delivered in-house with the support of the QUB Careers Service and external experts.

Compulsory Element

Attend at least five of the six lectures and complete successfully all the quizzes as well as the two workshops.

Assessment

Attend at least five of the six lectures, complete successfully the quizzes and attend and participate in the two workshops.

- **MTH2011 Linear Algebra (Sem 1)**

Pre-requisites: none

Lecturer: Dr D Barnes

Introduction

The techniques of linear algebra are core to most areas of mathematics, statistics, data analysis, physics and computer science. This module will build upon the level 1 linear algebra to develop the theory, methods and algorithms needed throughout the degree programme and beyond.

Content

- Recap and extend to fields such as C, the notions of abstract vector spaces and subspaces, linear independence, basis, dimension.
- Linear transformations, image, kernel and dimension formula.
- Matrix representation of linear maps, eigenvalues and eigenvectors of matrices.
- Matrix inversion, definition and computation of determinants, relation to area/volume.
- Change of basis, diagonalization, similarity transformations.
- Inner product spaces, orthogonality, Cauchy-Schwarz inequality.
- Special matrices (symmetric, hermitian, orthogonal, unitary, normal) and their properties.
- Basic computer application of linear algebra techniques.

Additional topics and applications, such as: Schur decomposition, orthogonal direct sums and geometry of orthogonal complements, Gram-Schmidt orthogonalization, adjoint maps, Jordan normal form.

Assessment

Continuous assessment 30%
Written examination 70%

- **MTH2012   Analysis (Sem 1)**

Pre-requisite: MTH1011 Introduction to Algebra and Analysis and MTH1021 Mathematical Methods 1

Lecturer: Dr A Zhigun

Introduction

This module extends and develops several core ideas from Level 1 analysis and seeks to deepen your understanding of them.
Its primary aim is to equip you with some key techniques of mathematical analysis, so that you can apply them both within mathematics itself and beyond in various other disciplines.

Content

Cauchy sequences, especially their characterisation of convergence. Infinite series: further convergence tests (limit comparison, integral test), absolute convergence and conditional convergence, the effects of bracketing and rearrangement, the Cauchy product, key facts about power series (longer proofs omitted). Uniform continuity: the two-sequence lemma, preservation of the Cauchy property (and the partial converse on bounded domains), equivalence with continuity on closed bounded domains, a gluing lemma, the bounded derivative test. Mean value theorems including that of Cauchy, proof of l'Hôpital's rule, Taylor's theorem with remainder. Riemann integration: definition and study of the main properties, including the fundamental theorem of calculus.

Assessment

Exam 70%
Class Test 20%
Assignments 10%

Please note the examination for this module will take place in January.

- **MTH2013 Metric Spaces (Sem 2)**

Pre-requisite: MTH2011 Linear Algebra, MTH2012 Analysis

Lecturers: Dr G Kiss

Introduction

Analysis in Semester 1 is the study of convergence and continuity, it is fundamentally linked to the structure of the real numbers. The aim of this module is to move gradually away from real numbers to the more general setting of metric spaces. A metric space is a set with a notion of distance, called a metric. The most familiar example is the real line, with the distance from x to y given by $|x - y|$. This notion allows us to define convergence and continuity in a much more abstract setting, and induces topological properties, like open sets and closed sets which lead to the study of more abstract topological spaces.

Content

- definition and examples of metric spaces (including function spaces)
- open sets, closed sets, closure points, sequential convergence, density, separability
- continuous mappings between metric spaces
- completeness
- normed spaces
- Hilbert spaces

Assessment

Continuous assessment 30%
Written examination 70%

- **MTH2014 Group Theory (Sem 2)**

Pre-requisite: MTH1011 Introduction to Algebra and Analysis, MTH2011 Linear Algebra

Lecturer: Dr S Balchin

Introduction

Group Theory is the branch of Algebra concerned with the study of groups. It is quite different from Linear Algebra and its origins can be traced back to the work of Lagrange on permutation groups. Groups usually arise in connection with invariance properties of the objects under study, e.g., the collection of all geometrical transformations of the Euclidean plane that leave invariant certain figure on it, form a group. Applications of group theory can be found in almost every area of mathematics, and also in chemistry and physics. This module will introduce you to some of the main concepts and techniques of the subject.

Content

- definition and examples of groups and their properties
- countability of a group and index
- Lagrange's theorem
- normal subgroups and quotient groups
- group homomorphisms and isomorphism theorems
- structure of finite abelian groups
- Cayley's theorem
- Sylow's theorem
- composition series and solvable groups

Assessment

Continuous assessment 30%
Written examination 70%

- **MTH2021 Mathematical Methods 2 (Sem 2)**

Pre-requisite: MTH1011 Introduction to Algebra and Analysis, MTH1021 Mathematical Methods 1

Lecturers: Dr S Shkarin, Dr G Gribakin

Introduction

This module covers complex analysis, partial differential equations and some of the interconnections between them. Complex analysis is the calculus of functions of a complex variable. It has elegant and useful applications in other areas of mathematics, in the physical sciences and in engineering. One of its most powerful uses, which we will study in the module, is in the evaluation of large classes of real integrals that are very hard or even impossible to do in any other way. Partial differential equations are differential equations for functions of more than one variable, e.g., position and time. Most areas of physics hinge on partial differential equations. They also have key applications in other subjects such as financial mathematics and mathematical biology.

Content

Functions of a complex variable: limit in the complex plane, continuity, complex differentiability, analytic functions, Cauchy-Riemann equations, Cauchy's theorem, Cauchy's integral formula, Taylor and Laurent series, residues, Cauchy residue theorem, evaluation of integrals using the residue theorem.

Introduction to partial differential equations. Wave equation, heat equation and Laplace's equation. Separation of variables. Bessel functions. Fourier method. Fourier series and Fourier transforms.

Assessment

Assignment 15%
Project 25%
Exam 60%

- **MTH2031 Classical Mechanics (Sem 1)**

Pre-Requisites: MTH1021 Mathematical Methods 1

Lecturer: Dr G Gribakin

Introduction

Mechanics has always been a source of inspiration for mathematics and mathematicians. Calculus, as well as the famous Newton's laws, were “invented” by Newton largely because he wanted to solve a single but very important mechanical problem, the problem of planetary motion. Mechanics was at the beginning of such branches of mathematics as theory of functions, calculus of variations, differential equations and more recently, theory of chaos. Mechanics and its mathematical methods are important in optics, electromagnetic theory, statistical mechanics, quantum mechanics, theory of relativity, quantum field theory and many ‘non-physical’ applications such as theory of optimisation and control.

For students, besides giving a comprehensive picture of mechanical phenomena and teaching how to solve a wide variety of problems, Classical Mechanics offers a unique opportunity to see the mathematical methods they have learned at work and to practice their mathematical skills. Vectors, partial derivatives, single and multiple integrals, differential equations, stationary points, complex numbers, as well as matrices and determinants are all among the tools used in the course.

Content

- Introduction to calculus of variations.
- Recap of Newtonian mechanics.
- Generalised coordinates. Lagrangian. Least action principle. Conservation laws (energy, momentum, angular momentum), symmetries and Noether’s theorem. Examples of integrable systems. D’Alembert’s principle. Motion in a central field.
- Small oscillations and normal modes. Rigid body motion.
- Legendre transformation. Canonical momentum. Hamiltonian. Hamilton’s equations. Liouville’s theorem. Canonical transformations. Poisson's brackets.

Assessment

Exam 70%
Coursework 30%

Please note the examination for this module will take place in January.

- **SOR2002 Statistical Inference (Sem 2)**

Pre-requisites: SOR1020 Introduction to Probability and Statistics

Lecturer: Dr L McFetridge

Introduction

Data is everywhere, being collected in all application areas and creating a huge growth in demand for qualified statisticians and data scientists. As the Economist noted, “the world’s most valuable resource is no longer oil, but data”. This module will lay the foundations for harnessing this important resource. It is therefore a key SOR module that provides the skills for extracting information from data and thus is a pre-requisite for most SOR modules in Levels 3 and 4.

There is a need for data enabled decision making. This module provides students with the skills to be able to analyse and interpret real world data, focusing first on initial data analysis and leading on to the use of linear regression to analyse quantitative information. Due to the practical nature of statistics, reference will be given throughout the module as to how the theory and computer elements that are taught relate to real world problems. This provides a valuable insight into the applications of statistics in industry, allowing students to gain the necessary skills required for such a working environment.

To put these skills into practice, students will undertake a group project to analyse real world data, replicating a typical project undertaken in industry. The module covers data analysis, sample diagnostics, estimation, linear regression, experimental design, hypothesis testing, and Bayesian methods.

Content

- *Statistical Investigations*: Understanding the main stages undertaken within a statistical analysis.
- *Initial Data Analysis & Sample Diagnostics*: This is the first step in analysing any real-world data. It focuses on gaining a better understanding of the data and therefore which statistical method would be most appropriate to use in its analysis. Testing for independence, serial correlations, normality, etc. will be explored.
- *Linear Regression*: Following initial data analysis, statisticians utilise statistical models to extract information from complex data. One of the fundamental statistical models is a linear model. This model will be introduced, illustrated using real world data and validated using goodness-of-fit measures.
- *Experimental Design and Comparative Studies*: Statisticians are often asked to design the collection of data for studies, such as clinical trials. The principles of experimental design and advantages/disadvantages of different sampling schemes will be discussed.
- *Estimation of parameters*: This will explore the concepts of bias and efficiency in the estimation of unknown parameters. Relative efficiency, sufficiency and mean square error will be introduced. The Fisher-Neyman factorization theorem will be explored.

- *Maximum Likelihood*: This is one of the most common methods of estimation. Construction of the likelihood function, calculation of a corresponding estimate and its properties will be discussed.
- *Significance Tests and Hypothesis Testing*: Hypothesis tests are utilised extensively in the analysis of data. The origins of some commonly used equations will be demonstrated utilising the Neyman-Pearson lemma. The use of computer intensive methods will be discussed, including randomization tests and Monte Carlo sampling.
- *Bayesian Methods*: The demand for Bayesian estimation is rapidly increasing in recent years. Such methods can incorporate prior beliefs about the situation under analysis, making the process extremely appealing to analysts. Conjugate families, prediction and the use of improper and non-informative priors will be discussed.

Assessment

Exam 70%
Report 20%
Presentation 10%

- **SOR2003 Methods of Operational Research (Sem 1)**

Pre-requisite: SOR1020 Introduction to Probability and Statistics, SOR1021 Introduction to Statistical & Operational Research Methods

Lecturer: Dr S Moutari

Introduction

This course applies mathematical analysis to a series of problems which occur in business and industry. The analysis can be more far reaching if we use a deterministic model but a degree of uncertainty (e.g. about future events) is often an important feature of the situation and a stochastic model has to be used. The statistical knowledge assumed is that contained in SOR1001. Although novel ways of setting out the work may be used in some topics, the mathematical techniques required on this course are no more advanced than simple calculus and algebra and most practical problems require only arithmetic and the use of tables.

The aim of the course is to teach a range of simple techniques illustrating the application of mathematics and probability theory to the problems of business and industry. Apart from the first two chapters each chapter is a distinct and separate topic. Some topics (e.g., Forecasting) involve lengthy calculation and students are taught how to use a spreadsheet for the computation. Students who do not have access to a spreadsheet on a personal computer can use the Open Access Areas. Specific instructions on the use of the Excel spreadsheet is given on the course and there is a practical session in an Open Access Area.

Homeworks are an essential part of the learning process, but there is no continuous assessment.

Emphasis is placed on choosing the correct model for the circumstances and on presenting answers in a form intelligible to management. If a question is posed in words, then the final answer should be in words and not left in algebra or in a table. The practical problems associated with obtaining data are discussed. The answer should be to a number of significant figures consistent with the accuracy of the original data, or rounded to an integer if that is appropriate.

Content

Deterministic and stochastic inventory models; simple and adaptive forecasting; theory of replacement of equipment; quality control, acceptance sampling by attribute and variable; network planning including the use of PERT, LP, Gantt charts and resource smoothing; decision theory, including utility curves, decision trees and Bayesian statistics; simple heuristics.

Assessment

Exam 70%
Report 20%
Presentation 10%

Level 3 Modules

- **AMA3011 Applied Mathematics Project (Sem 1 or Sem 2)**

Pre-requisite: While there are no specific pre-requisites, this course is intended for students at stage 3 of BSc Mathematics, Applied Mathematics and Physics or Theoretical Physics programmes, and a mathematical knowledge and ability commensurate with this stage is assumed.

Module Co-ordinators: Dr G Gribakin (Sem 1), Dr D Green (Sem 2)

Introduction

This module (or, alternatively, PMA3013 or PHY3007) is compulsory for all students on a BSc pathway (apart from Mathematics with Finance) and constitutes a self-study project on an advanced mathematical topic under the supervision of a member of staff. The module will allow a student to study a particular topic in more depth than possible in a taught module. It also offers students the opportunity to see how the mathematical skills developed in the programme are used to investigate more extensive problems.

Students will be offered a choice of topic subjects which can span the entire range of applied mathematics and statistics, including some theoretical physics.

To further develop communication skills, the outcomes of the study will be presented through a written report and through a poster presentation. In the poster presentation, students will explain their work in a short presentation to members of staff, with a brief follow-up discussion on points of interest.

Contents

Self-study of an advanced mathematical topic under the supervision of a member of staff. Students will be offered a choice of subjects, which can span the entire range of applied mathematics, including theoretical physics. The study concludes with a written report and a poster presentation.

Assessment

Report 80%
Presentation 20%

- **AMA3020 Investigations (Sem 2)**

Pre-Requisites: While there are no specific pre-requisites, this course is intended for MMath/MSci students in Mathematics, Mathematics and Statistics & Operational Research, Applied Mathematics and Physics, Mathematics and Computer Science, and Theoretical Physics, and a mathematical knowledge and ability commensurate with this stage is assumed.

Lecturers: Dr A Brown, Dr M Gruening

Introduction

Problem-solving is a key skill in many domains, from finance to academia, from software development to data analysis. Being able to address accurately a previously unseen problem, finding creative manners to solve it, and presenting complex concepts in a simple and effective way are invaluable assets. This module will facilitate the development of problems-solving skills that are expandable in a broad domain of fields and areas.

This module is taken by MMath/MSci students intending to take AMA4005 or SOR4001 projects in Level 4.

Content

Students conduct a short practice investigation, followed by two short investigations (in small groups and solo) in a range of problems in Applied Mathematics and Theoretical Physics. This is followed by a long investigation, which is a literature study of a Mathematical or Theoretical Physics topic not covered in the offered (or chosen) modules. The two short and the long investigation are typed up in reports and submitted for assessment.

Assessment

Presentation 20%
Report 1 15%
Report 2 20%
Report 3 45%

- **AMA3022 Team Project: Mathematics with Finance (Sem 2)**

Pre-requisite: Only available to students on the BSc Mathematics with Finance programme.

Lecturers: Dr D Dundas

Introduction

This is a company-led Level 3 project module for students on the Mathematics with Finance Pathway. It is designed to develop commercial, technical and team-working skills. The companies involved will provide problems that they want investigated. After meeting with all the companies in the first week and getting an overview of all the projects on offer, students rank their preferred choice of project. Based on these rankings, teams are formed and a project is assigned to each team. The students then meet with the companies to get a fuller brief. Each team then produces team branding, carries out some detailed initial exploratory work and submits a proposal outlining what they aim to offer the company. After agreeing the proposed work with the company, the teams then work with their partner company to deliver the project.

Contents

Business skills workshop. Presentation skills. Negotiation skills. Customer relationship. Project management/team building. Teams required to negotiate, plan, develop and deliver a completed task working as a group, commissioned by the `client’ company. The project will require software development skills.

Assessment

Business Proposal 30%
Report 40%
Presentation 10%
Peer Evaluation (20%)

- **MTH3011 Measure and Integration (Sem 1)**

Pre-requisite: MTH2012 Analysis and MTH2013 Metric Spaces

Lecturers: Dr Y-F Lin

Introduction

The theory of integration, developed by Lebesgue in the early part of the twentieth century in the context of the real line and subsequently extended to more general settings, is indispensable in modern analysis. The Lebesgue theory allows a very wide class of functions to be integrated and includes powerful convergence theorems which are not available in Riemann integration. In this module the theory is developed in the context of a general σ-algebra of sets. Special attention is given to the case of Lebesgue measure on the reals, and some applications of the integral to Fourier series are given.

Content

σ-algebras of sets, measurable spaces, measurable functions. Measures. Integrals of non-negative measurable functions: properties including Fatou's lemma and monotone convergence theorem. Integrable functions: Lebesgue dominated convergence theorem. Lebesgue integral on intervals: comparison with Riemann integral. L^p-spaces: inequalities of Hölder and Minkowski; Fourier series in L^2.

Assessment

Assignments 30%
Exam 70%

Please note the examination for this module will take place in January.

- **MTH3012 Algebra (Sem 1)**

Pre-requisite: MTH2011 Linear Algebra and MTH2014 Group Theory

Lecturer: Dr D Barnes

Introduction

The purpose of this module is to give a general introduction to the theory of algebra, which is a generalisation of linear algebra. The central topic is the concept of ring: a set with an addition and a multiplication. Rings include the most familiar number systems: integers, rationals, real numbers and complex numbers. Moreover, the notions of matrix multiplication and polynomial multiplication neatly fit into the theory of rings. In this module we will study both the general theory of rings and specific properties of the rings of most interest to us: the ring of integers, polynomial rings and matrix rings. This material is also the starting point for (algebraic) number theory and its primary application: cryptology.

Content

Rings, subrings, prime and maximal ideals. Quotient rings, homomorphisms, and isomorphism theorems. Integral domains, Euclidean domains and principal ideal domains. Ring theoretic properties of the integers, modulo arithmetic and elementary number theory. Polynomial rings, irreducibility and factorisation. Further examples of rings and algebraic structures.

Assessment

Continuous assessment 30%
Written examination 70%

- **MTH3021 Dynamical Systems (Sem 2)**

Pre-requisite: MTH2011 Linear Algebra, MTH2013 Metric Spaces

Lecturer: Dr G Kiss

Introduction

A dynamical system is a mathematical description of a real-world process evolving in time, such as the number of infectious individuals in a population during an outbreak, the price of a financial instrument, or the positions and velocities of finite point masses. The mathematical representation of a time dependent phenomenon can be given in terms of differential equations or iterated functions, and very often models are non-linear. In this module, by studying the existence and qualitative properties of invariant objects such as equilibria, closed orbits and their invariant manifolds, we will learn how a well-formulated rather abstract mathematical model can be used, with mathematical rigour, to control an epidemic, predict the unpredictable and obtain low-energy interplanetary spacecraft trajectories.

Contents

Continuous dynamical systems
- Fundamental theory: existence, uniqueness and parameter dependence of solutions;
- Linear systems: constant coefficient systems and the matrix exponential; nonautonomous linear systems; periodic linear systems.
- Topological dynamics: invariant sets; limit sets; Lyapunov stability.
- Grobman-Hartman theorem.
- Stable, unstable and centre manifolds.
- Periodic orbits: Poincare-Bendixson theorem.
- Bifurcations
- Applications: the Van der Pol oscillator; the SIR compartmental model; the Lorenz system.

Discrete dynamical systems
- One-dimensional dynamics: the discrete logistic model; chaos; the Cantor middle-third set.

Assessment

Continuous assessment 30%
Written examination 70%

- **MTH3023 Numerical Analysis (Sem 1)**

Pre-requisite: MTH2011 Linear Algebra

Lecturer: Dr D Dundas

Introduction

In many real-world applications of mathematics, it may not be possible to obtain analytical solutions. Numerical Analysis is concerned with devising methods for finding approximate, numerical solutions to mathematically expressed problems. These methods can be analysed for their accuracy, efficiency and robustness. For example, some methods will guarantee convergence to a solution, but may require much effort, while other methods may converge quickly with less effort, but may also diverge. Faced with such differing behaviour of the methods, we need to be able to determine the 'best' strategy to adopt for a given problem. In MTH3023 we cover the basic introductory material of Numerical Analysis. We investigate the solution of equations, interpolation, function approximation, differentiation, integration and the solution of ordinary differential equations.

Content

Introduction and basic properties of errors: Introduction; Review of basic calculus; Taylor's theorem and truncation error; Storage of non-integers; Round-off error; Machine accuracy; Absolute and relative errors; Richardson's extrapolation.
- Solution of equations in one variable: Bisection method; False-position method; Secant method; Newton-Raphson method; Fixed point and one-point iteration; Aitken's "delta-squared" process; Roots of polynomials.
- Solution of linear equations: LU decomposition; Pivoting strategies; Calculating the inverse; Norms; Condition number; Ill-conditioned linear equations; Iterative refinement; Iterative methods.
- Interpolation and polynomial approximation: Why use polynomials? Lagrangian interpolation; Neville's algorithm; Other methods.
- Approximation theory: Norms; Least-squares approximation; Linear least-squares; Orthogonal polynomials; Error term; Discrete least-squares; Generating orthogonal polynomials.
- Numerical quadrature: Newton-Cotes formulae; Composite quadrature; Romberg integration; Adaptive quadrature; Gaussian quadrature (Gauss-Legendre, Gauss-Laguerre, Gauss-Hermite, Gauss-Chebyshev).
- Numerical solution of ordinary differential equations: Boundary-value problems; Finite-difference formulae for first and second derivatives; Initial-value problems; Errors; Taylor-series methods; Runge-Kutta methods.

Assessment

Project 1 20%
Project 2 30%
Exam 50%

- **MTH3024 Modelling and Simulation (Sem 2)**

Pre-requisite: MTH2011 Linear Algebra

Lecturers: Prof H van der Hart, Dr L Stella

Introduction

Mathematical modelling consists in translating a phenomenon observed in the real world into a mathematical framework, often involving associated equations. Only rarely can an analytical solution for these equations be found and instead numerical simulations are required. Numerical simulations are calculations run on a computer according to a program that implements the mathematical model. Mathematical models and numerical simulations are used extensively. Examples are numerous: Major building design must enable rapid emergency evacuation in line with legislation. Sustainable city design requires the modelling of traffic, including vehicles, pedestrians and cyclists. Understanding the green impact of policy decisions requires mathematics to estimate the effects of policy decisions and understanding how these effects impact the environment.

Many of the taught modules introduce particular mathematical models. However, these models are often applied to ideal systems, so that analytic results can be obtained. Most real systems are not ideal systems, however. Modelling and simulation then aims to identify suitable approximations to capture the essence of the real system, so that appropriate outcomes can be obtained. This module aims to introduce students to the modelling and simulation process through hands-on experience.

Content

In this module, students will analyse real-life situations, build a mathematical model, solve it using analytical and/or numerical techniques, and analyse and interpret the results and the validity of the model by comparing to actual data. The emphasis will be on the construction and analysis of the model rather than on solution methods. Two group projects will fix the key ideas and incorporate the methodology. This will take 6-7 weeks of term and will be supported with seminars and workshops on the modelling process. Then students will focus on a final project, ideally with real-life application, and work on this for the remaining weeks of term. They will present their results through a variety of approaches. A key element of the module is the discussion of ideas. These discussions can take place not only within a group, but also between groups. This highlights that the same type of problem can often be approached by entirely different means.

The starting group project will be focused and offer a limited number of specific modelling problems. The second project will also be focused, but offer more scope for students to identify their own specific follow-on development. A wider range of projects will be available for the final project, spanning a wide range of disciplines in which mathematical modelling plays a critical role. The projects will place a significant and increasing emphasis on students' own initiative.

Assessment

Project work 100%

At least 66% participation rate in computer practicals associated with each of the assessed components is required. If this participation rate is not achieved, without a valid reason, a mark of 0 will be awarded for the associated component.

- **MTH3025 Financial Mathematics (Sem 2)**

Pre-requisite: While there are no specific pre-requisites, this course is intended for students at stage 3 of either an MMath/MSci or a BSc Mathematics pathway, and a mathematical knowledge and ability commensurate with this stage is assumed.

Lecturer: Dr A Hodgkinson

Introduction

Mathematical skills are highly sought after in the financial services industries, and this employment sector remains a favoured destination for graduates. Around 40% of Mathematics graduates entering employment across the UK (see www.prospects.ac.uk for recent data) go into financial services, which includes, accountancy, retail and investment banking, mergers and acquisitions, insurance and actuarial work, capital market trading, and hedge fund employment, and so on.

At the low end of this sector, retail banking for example, a degree in mathematics is certainly not essential. This work is mainly concerned with simple arithmetic operations. However, at the high end of financial services, in a hedge fund for example, employers expect to see PhD-level qualifications in mathematics from their applicants along with excellent software skills. These mathematicians are involved in the business of derivative pricing and trading and earn salaries well over 100k. Derivatives are financial products (instruments as they are called in the trade) derived from assets that have an unpredictable price. The total outstanding notional value of derivatives contracts today has grown beyond a quadrillion dollars (that's 10^{15} to you and me). It is a perilous and lucrative business!

Derivatives were originally devised to avoid risk by providing an insurance on a risky asset. Nowadays, they are an essential part of risk taking in capital markets. Indeed the speculation in buying and selling these instruments, specifically credit derivatives, precipitated the current credit crunch. Of course, this trade relies upon knowing the fair price of a derivative. Pioneering work by Black, Merton and Scholes, showed that, under certain assumptions for the unpredictability of the asset, the price of the derivative obeys a partial-differential equation. The construction of such equations and their solution is where mathematicians come in!

The objective of the course is to provide an introduction to the mathematical techniques which can be applied to pricing problems for financial derivatives. Specifically, our focus is on stochastic calculus and the theory and practice of pricing simple derivatives such as contracts and options.

Content

Introduction to financial derivatives: forwards, futures, swaps and options; Future markets and prices; Option markets; Binomial methods and risk-free portfolio; Stochastic calculus and random walks; Ito's lemma; the Black-Scholes equation; Pricing models for European Options; Greeks; Credit Risk.

Assessment

Presentation 10%
Report 20%
Exam 70%

- **MTH3031 Classical Fields (Sem 1) [SUSPENDED]**

Pre-requisite: MTH2031 Classical Mechanics or PHY2004 Electricity, Magnetism & Optics

Lecturer:

Introduction

This module develops further the fundamental ideas that enable mathematics to describe the Universe that was started in Classical Mechanics. It introduces the notion of 4-dimensional space-time and Einstein's relativity and Lorentz covariance as the main principles that the theory must obey. It then uses the Least Action Principle to build the description of the motion of charged particles and electron-magnetic fields and their interaction. Having set up the equations for the electro-magnetic field in covariant, 4-dimensional form, we then "descend" into the more familiar 3+1 (space + time) form to derive Maxwell's equations and study their solutions in a range of context, from electrostatics and magnetostatics to waves. The ideas and description of classical (electro-magnetic) fields developed in this module can lead to further generalisation in the form of Einstein's general relativity. When combined with Quantum Theory, this allows the development of the theory of Quantum Fields which underpins all modern theories of elementary particles.

Content
- The principle of least action and conservation laws in Classical Mechanics (recapping).
- 4-dimensional space time, interval, 4-vectors, tensors, Lorentz covariance.
- Action and Lagrangian for a particle, energy and momentum.
- 4-potential and the Lagrangian for a charged particle in an electromagnetic field, relativistic equation of motion and Lorentz's force, electric and magnetic fields.
- Lagrangian of the electromagnetic field, Maxwell's equations in covariant form, charge density and current density, continuity equation, Maxwell's equations in conventional (3+1) form, and in integral form.
- Electrostatics (general ideas, Coulomb's law, fields of various charge distributions, conductors, method of images, electric dipole moment).
- Magnetostatics (general ideas, Biot-Savart-Laplace law, fields of systems of currents, magnetic dipole moment).
- Electromagnetic waves, plane wave, polarisation, monochromatic wave.
- Electromagnetic radiation: retarded potentials, dipole radiation (electric, magnetic), Larmor formula.

Assessment

Assignment 1 15%
Assignment 2 15%
Exam 70%

- **MTH3032 Quantum Theory (Sem 1)**

Pre-requisite: MTH2031 Classical Mechanics or PHY2001 Quantum and Statistical Physics, and MTH2021 Mathematical Methods 2

Lecturer: Dr L Stella

Introduction

Quantum mechanics brought about the most fundamental change in our understanding of how the world works. It describes the behaviour of microscopic particles (electrons, photons, atomic nuclei, etc.) and their interactions in a way that is very different from our everyday experience. Key points of the theory are wave-particle duality (e.g., the ability of particles to display typical wave-like properties, such as interference), quantisation (i.e., restriction of possible values of some physical observables, such as energy or angular momentum, to a discrete set of values), statistical nature of its predictions, and the role played by the observer in a measurement process.

In this module we develop the mathematical methods that enable one to describe how nature behaves at small scales, e.g., that of individual atoms. The mathematical setting of Quantum Theory is that of Hilbert spaces and linear operators, and its practical aspects involve dealing with matrices, eigenvalue problems and differential equations.

Students completing this module will understand the basic principles of quantum mechanics and its mathematical tools, learn how to solve a few simple, fundamental problems, and practice approximate methods that greatly widen the range of problems that can be solved.

Quantum Theory builds on Classical Mechanics and open the route to Quantum Fields which is the framework for modern theories of elementary particles.

Content

- Overview of classical physics and the need for new theory.
- Basic principles: states and the superposition principle, amplitude and probability, linear operators, observables, commutators, uncertainty principle, time evolution (Schrödinger equation), wavefunctions and coordinate representation.
- Elementary applications: harmonic oscillator, angular momentum, spin.
- Motion in one dimension: free particle, square well, square barrier.
- Approximate methods: semiclassical approximation (Bohr-Sommerfeld quantisation), variational method, time-independent perturbation theory, perturbation theory for degenerate states (example: spin-spin interaction, singlet and triplet states).
- Motion in three dimensions: Schrödinger equation, orbital angular momentum, spherical harmonics, motion in a central field, hydrogen atom.
- Atoms: hydrogen-like systems, Pauli principle, structure of many-electron atoms and the Periodic Table.

Assessment

Assignments 30%
Exam 70%

- **MTH3099 Placement Year Out (Full year)**

Pre-requisite: MTH2010. Placement documentation (including contract, job description and health & safety agreement) that has been approved by the School must be in place.

Lecturer: Dr S Moutari

Introduction

This is a 120 CAT point module (equivalent to a year of academic study) that is taken by students who are taking an approved placement year as part of their degree. As part of the placement, students keep a diary of work undertaken and use this to produce a portfolio at the end of the placement. This portfolio will allow students to highlight the work undertaken and to reflect on the skills developed.

Content

Course contents is as defined by the School-approved student contract and job description. A mid-placement visit by School staff will take place to ensure that the job role and conditions of employment reflect those described in the job description and contract.

Compulsory Element

Submission of all required paperwork associated with the placement.

Assessment

A placement portfolio to be submitted at the end of the placement. This portfolio is assessed on a Pass/Fail basis. The placement year does not contribute to your degree classification and therefore does not affect your progression onto the next year of study.

- **PMA3013 Mathematical Investigations (Sem 2)**

Pre-Requisites: None

Lecturer: Dr S Shkarin

Introduction

This module is an alternative to AMA3011 as a compulsory project module in Level 3 for BSc students (apart from Mathematics with Finance). It is also taken by the MMath/MSci students who intend to take PMA4001 Project in their final year.

Content

This module is concerned with the investigation processes of mathematics, including the construction of conjectures based on simple examples and the testing of these with further examples, aided by computers where appropriate. A variety of case studies will be used to illustrate these processes. A series of group and individual investigations will be made by students under supervision, an oral presentation will be made on one of these investigations. While some of the investigations require little more than GCSE as a background, students will be required to undertake at least one investigation which needs knowledge of Mathematics at Level 2 or Level 3 standard and/or some background reading.

Assessment

Solo Project 50%
Group Project 40%
Presentation 10%

- **SOR3004 Linear Models (Sem 1)**

Pre-requisite: SOR2002 Statistical Inference

Lecturer: Dr H Mitchell

Introduction

The aim of this module is to cover linear models encompassing multiple linear regression and analysis of variance (ANOVA). These models are the workhorses of statistical data analysis and are found in virtually all branches of the sciences as well as in the industrial and financial sectors.

Multiple linear regression is concerned with modelling a measured response as a function of explanatory variables. For example, a pharmaceutical company might use a a regression model to relate the effectiveness of a new cancer drug to the patients age, gender, weight, diet, tumour size, etc. ANOVA is concerned with the analysis of data from designed experiments. A materials manufacturer for example, may wish to analyse the results from an experiment to compare the heat resisting properties of four different polymers.

Regression and ANOVA will be initially developed using a classic least squares approach and later the correspondence between least squares and the method of maximum likelihood will be examined. After a thorough development of linear models the groundwork will have been laid to allow an extension to the broader class of Genealized Linear Models (GLM). These permit regression models to be applied to situations where the recorded response is not normally distributed. One famous example of the use of GLM was the analysis of O-ring failures on the space shuttle Challenger.

An important element of this module will be a weekly practical data analysis class using the SAS software package. SAS is probably the leading statistical package used in industry. These classes, lasting up to three hours, will introduce the student to elementary data entry in SAS, elementary matrix manipulation using the SAS Interactive Matrix Language (IML) and analysis of data using linear and generalized linear models.

Contents

Linear regression. Non-singular case: analysis of variance, extra sum of squares principle, generalised least squares, residuals. Singular case: generalised inverse solution, estimable functions. Experimental designs: completely randomised, randomised block, factorial; contrasts, analysis of covariance; Generalised linear model (GLM): maximum likelihood and least squares; exponential family; Poisson and logistic models; model selection for GLM.

Assessment

Exam 70%
Coursework 20%
Presentation 10%

Please note the examination for this module will take place in January.

- **SOR3008 Statistical Data Mining with Machine Learning (Sem 2)**

Pre-requisite: SOR3004 Linear Models

Lecturer: Dr K Cairns

Introduction

In the 1990's there was an explosive growth in both the generation and collection of data due mainly to the advancement of computing technology in processing and storage of data and the ease of scientific data collection. As a result, overwhelming mountains of data are being generated and stored. For example, in the business world large supermarket chains such as Wal-Mart and Sainsbury's collect data amounting to millions of transactions per day. In the US all health-care transactions are stored on computers yielding terabyte databases which are constantly being analysed by insurance companies. There are huge scientific databases as well. Examples include the human genome database project and NASA's Earth Observatory System. This has brought about a need for vital techniques for the modelling and analysis of these large quantities of data: data mining.

Data Mining is the process of selection, exploration, and modelling of large quantities of data to discover previously unknown regularities or relations with the aim of obtaining clear and useful results for the owner of the database. The application of data mining includes many different areas, such as market research (customer preferences), medicine, epidemiology, risk analysis, fraud detection and more recently within bioinformatics for modelling DNA.

This module will focus on data mining techniques which have evolved from and are strongly based on statistical theory.

Content

Introduction to Data Mining and Machine Learning; Supervised and Unsupervised Learning; Exploratory Data Analysis including Principal Component Analysis, Multiple Imputation; Cluster analysis; Classification including Decision tree analysis, Bayesian Networks, Probabilistic/Regression-Based Modelling; Prediction including Regression trees, Random Forests, Neural nets, Support Vector Machines; Association Rule Mining.

Assessment

Coursework 1 20%
Coursework 2 20%
Exam 60%

- **SOR3012 Stochastic Processes and Risk (Sem 1)**

Pre-requisite: SOR1020 Introduction to Probability & Statistics

Lecturer: Dr G Tribello

Introduction

The module teaches you about time series of random variables. A very practical approach to the subject is taken so you learn about these random time series by writing computer programs to generate and analyse random data. The final assessment of the module requires you to use what you have learned about modelling random processes to write a report for a real community transport organisation that is based in Fermanagh.

Contents

Logic and Boolean algebra, counting and combinatorics, set algebra, inclusion-exclusion theorem, mutually exclusive events, De Morgan Laws.

Axioms of probability, events and probability spaces, sigma-field, random variables, conditional probability, and expectation, Bayes' theorem, discrete and continuous random variables, moments and moment generating function. Laws of large numbers and central limit theorem.

Pairs of random variables, marginal probabilities, Cauchy-Schwartz Inequality in statistics, correlation and covariance.

Discrete time Markov chains, Chapman-Kolmogorov relation, limiting behaviour, transient, recurrent states and periodic states, limiting stationary distribution, hitting times and hitting probabilities.

Continuous time Markov chains, Kolmogorov forward equations, stationary distribution for continuous time Markov chains, Poisson process, MM1 Queue, inhomogeneous Poisson process and compound Poisson process.

Assessment

Exam 45%
Coursework 55%

- **MTH4311 Functional Analysis (Sem 2) [offered in 2025/26]**

Click here

- **MTH4321 Fourier Analysis and Applications to PDEs (Sem 2) [offered in 2024/25]**

Click here

- **MTH4322 Topological Data Analysis (Sem 1) [offered in 2025/26]**

Click here

- **MTH4323 Geometry of Optimisation (Sem 1) [offered in 2024/25]**

Click here

- **MTH4331 Quantum Fields (Sem 2) [SUSPENDED]**

Click here

- **MTH4332 Statistical Mechanics (Sem 2) [offered in 2025/26]**

Click here

Level 4 Modules

- **AMA4005 Project (Full year)**

Pre-requisite: This two-semester-long double module is only available to students on the MMath Mathematics or MSci Applied Mathematics and Physics/Theoretical Physics/Mathematics and Computer Science programmes.

Co-ordinator: Dr G De Chiara

Introduction

The project involves a substantial investigation of a research problem incorporating literature survey, development of appropriate theoretical models and, when necessary, the construction of computer programs to solve specific stages of the problem. The assessment consists in an oral presentation and a written dissertation. Each student will work under individual supervision of a member of staff.

Content

The mathematical contents of the project will depend on the nature of the research problem.

Assessment

Dissertation 80%
Presentation 20%

- **MTH4011 Topology (Sem 1)**

Pre-requisite: MTH3011 Measure and Integration

Lecturer: Dr A Blanco

Introduction

Topology (rather like Algebra or Analysis) is not so much a single branch of mathematics but a loose confederation of subject areas differing widely in their origins, techniques and motivation but united by sharing a common core of basic concepts and constructions. Problems of a topological nature include: how can we describe and classify knots? how can we describe and classify surfaces? to what extent is it possible to extend the ideas of analysis into sets that don't have metrics defined on them? what can be meant by saying that two objects are “fundamentally the same shape”, and how do we decide whether they are or not? what ‘models’ are available to describe certain aspects of theoretical computer science? Rather than attempting to supply answers to any such major questions, this module will concentrate on developing enough of the ‘common core’ to allow students to begin to appreciate how such issues can be tackled topologically.

Content

- Definition and examples (natural, geometric and pathological)
- Continuity and homeomorphisms
- Compact, Connected, Hausdorff
- Subspaces and product spaces
- Introduction to homotopy, calculations and applications

Assessment

Continuous assessment 30%
Written examination 70%

- **MTH4021 Applied Algebra and Cryptography (Sem 2)**

Pre-requisite: MTH3012 Rings and Modules

Lecturer: Dr T Huettemann

Introduction

Any two numbers can be added, subtracted and multiplied, and in many cases divided, to produce a new number. This is very familiar for real numbers, or for complex numbers, or for rational numbers. However, there are more general “number systems” that behave “like the reals” in most respects. Some of these number systems (or to use the technical term: fields) have only finitely many members, which may come as a surprise initially.

These finite fields are fascinating from a theoretical point of view, but also have important practical applications, for example in the area of cryptography (as used in “secure” internet connections nowadays, and by spies throughout all history). One of the reasons is that finite fields, as opposed to the field of real numbers, lend themselves to efficient calculations on computers.

Many applications are built on the theory of polynomials. After developing the foundational material to some extent, we will construct finite fields and discuss various applications; this may include encryption algorithms or generation of pseudo-random numbers.

Content

- (finite) fields and rings of polynomials over them.
- the division algorithm and splitting of polynomials.
- ideals and quotient rings, (principal) ideal domains, with examples from rings of polynomials.
- polynomials in several indeterminates, Hilbert’s basis theorem.
- applications of algebra to cryptography (such as affine Hill ciphers, RSA, lattice cryptography, Diophantine equations).
- optional topics may include Euclidean rings, unique factorisation domains, greatest common divisor domains.

Assessment

Continuous assessment 20%
Written examination 80%

- **MTH4022 Information Theory and Biodiversity (Sem 2)**

Pre-requisite: None

Lecturer: Prof H van der Hart

Introduction

Information theory concerns itself with the mathematics of communication. It underpins mobile phone technology, digital broadcasting, and many other aspects of modern technology, and as such is fundamental to current society. Information theory is a relatively new topic in mathematics, with many of the fundamental ideas formulated between 1940 and 1955. However, significant breakthroughs in communication theory were achieved far more recently: the most modern schemes for communication only started their development in the 1990s.

Information theory thus forms an important basis of the development of information technology in the 21st century. It is an area of mathematics with direct applications, and it is an area where mathematical ideas can be taken up very quickly in new technology. These mathematical ideas can be based on any area of mathematics, and so information theory is connected to a wide range of mathematics. But information theory is not merely an application of mathematics. Information theory has had an important and significant influence on mathematics and has given profound contribution to pure mathematics.

Information transfer occurs through so-called "codes". These codes can take various forms, ranging from the English language to the Morse code to the ASCII code for computer data storage. In this module, we will define what we mathematically mean with a code. We will then develop measures for the amount of information stored within a message, and introduce the key quantity of information theory, entropy. We will investigate how communication affects entropy, leading to the fundamental theorem of information theory, relating information transfer and communication capacity.

We will conclude with the application of information theory to measurement of diversity. Biodiversity is an obvious and important example, but the same principles can also be used to measure economic diversity or social diversity. The measurement of biodiversity introduces new mathematical questions, which have only been investigated within the last 15 years. Do two similar, but different, species from the same genus have the same level of diversity as two species from different genus? So we will investigate how we can change the information theory, correct for communication, to modern mathematical questions of relevance to biology.

The module combines both mathematical theory and application. This interplay is important. We need to have mathematically robust measurements, but these measurements also need to be relevant to practitioners in application areas. The module therefore highlights both how mathematical theory is developed from underpinning principles, and how this theory can then be applied in a practical context.

Content

Introduction to information theory. Basic modular arithmetic and factoring. Finite-field arithmetic. Random variables and some concepts of probabilities. RSA cryptography and factorisation. Uniquely decipherable and instantaneous codes. Optimal codes and Huffman coding. Code extensions. Entropy, conditional entropy, joint entropy and mutual information. Shannon noiseless coding theorem. Noisy information channels. Binary symmetric channel. Decision rules. The fundamental theorem of information theory. Correspondence between diversity and information. Effective biodiversity numbers. Transformations of entropy and diversity profiles. Inclusion of species similarity.

Assessment

Exam 70%
Report 30%

- **MTH4023 Mathematical Methods for Quantum Information Processing (Sem 2)**

Pre-requisite: MTH2011 Linear Algebra

Lecturers: Dr G De Chiara

Introduction

We are currently witnessing an information revolution: digital devices are everywhere around us. If the incredible level of miniaturisation of electronic devices continues at the current pace, in a few years the elementary components will be made of a few atoms. At this level, physical effects ruled by quantum mechanics will start playing a major role. Inspired by this change of perspective, quantum information processing has been developed as a new framework for future computers. The logic of these quantum computers is different from the traditional computers, as the elementary unit of information, the quantum bit, can be in a superposition of two states 0 and 1.

The aim of this module is to develop the mathematical theory that underpins most application of this emerging field, including quantum computing, quantum communication (teleportation), and entanglement. Quantum mechanics is not a requisite and will be introduced at the beginning of the module as an abstract framework in linear algebra.

Students completing this module will acquire knowledge of the mathematical concepts of quantum information processing with possible applications in theoretical physics, applied, pure maths, and computer science.

Content

- Operatorial quantum mechanics: review of linear algebra in Dirac notation; basics of quantum mechanics for pure states.
- Density matrix and mixed states; Bloch sphere; generalised measurements.
- Maps and operations: complete positive maps; Kraus operators.
- Quantum Communication protocols: quantum cryptography; cloning; teleportation; dense coding.
- Quantum computing: review of classical circuits and logic gates; quantum circuits and algorithms; implementation of quantum circuits on small prototypes of quantum computers (IBM Quantum Experience); examples of physical Hamiltonians implementing quantum gates.
- Theory of entanglement: basic notions and pure-state entanglement manipulation; detection of entanglement; measures of entanglement; entanglement and non-locality, Bell's inequality; multipartite entanglement.

Assessment

Exam 70%
Project 30%

- **MTH4024 Practical Methods for Partial Differential Equations (Sem 1)**

Pre-requisite: MTH2021 Mathematical Methods 2

Lecturer: Dr C Ballance

Introduction

Many problems in applied mathematics reduce to solving either ordinary or, more usually, partial differential equations subject to certain boundary conditions. In many cases of practical interest exact analytical solutions are not available.

In non-relativistic quantum mechanics for example the problems involve finding numerical solutions to the Schrödinger equation. In fluid mechanics solutions of the Navier-Stokes equation are needed in the context for example of weather systems, or of hypersonic flow around a space shuttle, or the flow of blood through arteries. In financial mathematics solutions of the Black-Scholes equation that describes the pricing of options are required. A familiar partial differential equation arising in many problems in physics and engineering is the diffusion equation, which describes how heat flows from hot to cold regions and many other processes. Another familiar example is the wave equation that governs phenomena all around us: electromagnetic waves, the vibrations of guitar strings, and the propagation of the sound they produce.

In order to solve these or other problems we need to understand first the conditions that give rise to a unique solution. Since analytical solutions are only rarely available, we examine numerical techniques such as finite difference and finite element to solve particular problems. Creating these programs that solve multi-dimensional problems subject to specific boundary conditions is a large aspect of the course.

Content

- Basics: solving first order ordinary differential equations, partial derivatives, surface, volume and line integrals, the Gauss theorem, Stokes' Theorem.
- Partial differential equations (PDE) and their relation to physical problems: heat conduction, flow of a liquid, wave propagation, Brownian motion.
- First order PDE in two variables: the method of characteristics, the transversality condition, quasilinear equations and shock waves, conservation laws, the entropy condition, applications to traffic flows.
- Second order linear PDEs: classification and canonical forms.
- The wave equation: d`Alembert’s solution, the Cauchy problem, graphical methods.
- The method of separation of variables: the wave and the heat equations.
- Numerical methods: finite differences, stability, explicit and implicit schemes, the Crank-Nicolson scheme, a stable explicit scheme for the wave equation.
- Practical: the students are offered to solve a heat and a wave equation using the method of separation of variables and a finite difference scheme.
- The Sturm-Liouville problem: a theoretical justification for the method of separation of variables. Simple properties of the Sturmian eigenvalues and eigenfunctions.
- Elliptic equations: the Laplace and Poisson equations, maximum principles for harmonic

functions, separation of variables for Laplace equation on a rectangle.
- Green's functions: their definition and possible applications, Green's functions for the Poisson equation, the heat kernel.

Assessment

Exam 70%
Report 30%

- **MTH4031 Advanced Quantum Theory (Sem 1)**

Pre-requisite: MTH3032 Quantum Theory or PHY3001 Quantum Mechanics and Relativity

Lecturers: Dr M Gruening

Introduction

Quantum mechanics is one of the greatest intellectual achievements of the last century. In this course we will recap the mathematical framework of quantum mechanics, and show how it describes the structure of atoms and their collisions. Atomic, molecular and optical physics is a burgeoning field, with numerous Nobel prizes awarded in the area in the last 20 years. Intense research is ongoing internationally, aimed at obtaining ever more precise understanding of atomic structure and atomic interactions, and how they can be controlled by external fields. Such understanding is key to, e.g., developing new quantum-based technologies for metrology and cryptography, for precision tests of fundamental laws of physics, for the study of astrophysical phenomena, and for many other applications.

Students completing this module will expand the toolkit of mathematical methods of quantum mechanics, and obtain basic theoretical knowledge of atomic structure and introductory many-body quantum physics.

Content

- Review of fundamental quantum theory (Postulates of quantum mechanics; Dirac notation; Schrödinger equation; spin-1/2 systems; stationary perturbation theory).
- Coupled angular momenta: spin-1/2 coupling; singlet and triplet subspaces for two coupled spin-1/2 particles; Coupling of general angular momenta;
- Spin-orbit coupling; fine and hyperfine structures of the hydrogen atom.
- Time-dependent perturbation theory.
- Elements of collisions and scattering in quantum mechanics.
- Identical particles and second quantisation; operators representation.
- Basics of electromagnetic field quantisation.
- Systems of interacting bosons: Bose-Einstein condensation and superfluidity.

Assessment

Exam 80%
Presentation 20%

- **MTH4311 Functional Analysis (Sem 2) [offered in 2025/26]**

Pre-requisite: MTH2013 Metric spaces and MTH3011 Measure and Integration

Lecturer:

Introduction

At the beginning of the 20th century, the new discipline of Functional Analysis grew out of joint efforts by mathematicians from various countries to unify their understanding of *spaces of functions* and *spaces of operators*. In conjunction with the emerging area of *Topology*, novel tools were developed that allowed for the treatment of infinite dimensional vector spaces, which was hitherto out of reach. It quickly became apparent that this new area of Pure Mathematics was fundamental for many parts of modern Physics (such as Quantum Theory, for example) and many other sections of mathematics. For instance, in the study of partial differential equations it provides indispensable techniques.
This module introduces and discusses in detail the fundamental concepts of (linear) Functional Analysis such as Banach spaces and their (bounded) operators. As there is a strong research group in Functional Analysis at QUB, the module also provides the basis for further academic work (as part of a PhD, for example).

Content

A characterisation of finite-dimensional normed spaces; the Hahn-Banach theorem with consequences; the bidual and reflexive spaces; Baire's theorem, the open mapping theorem, the closed graph theorem, the uniform boundedness principle and the Banach-Steinhaus theorem; weak topologies and the Banach-Alaoglu theorem; spectral theory for bounded and compact linear operators.

Assessment

Assignment 1: 15%
Assignment 2: 15%
Exam 70%

- **MTH4321 Fourier Analysis and Applications to PDEs (Sem 2) [offered in 2024/25]**

Pre-requisites: MTH3011 Measure and Integration; taking MTH3021 Dynamical Systems is recommended.

Lecturer: Dr A Zhigun

Introduction

This module is dedicated primarily to the study of the Fourier series (FS) and Fourier transform (FT) in various functional spaces, including spaces of distributions. (The theory of distributions, or generalised functions, was developed by Sergei Sobolev and Laurent Schwartz in the 1930s and 1940s.) In your previous studies, you saw some of these tools being an integral part of effective methods to derive explicit solutions for certain partial differential equations (PDEs). This module extends and develops further the core ideas of such approaches and seeks to deepen your understanding of them.

Being useful in the analysis of PDEs as well as other applications, e.g., signal processing, properties of the FT and the FS are also of independent interest to mathematicians and constitute an elegant and well-developed theory that we are going to explore.

This course relies heavily on key facts about the Lebesgue integral and serves as an important application of the measure and integration theory.

Content

Fourier transform:
- Fourier transform and its main properties in L^1, Schwartz space, and L^2
- convolution theorem
- inverse Fourier transform

Distributions:
- space of test functions, distributions, basic examples (regular distributions, Dirac delta)
- main operations (multiplication by functions, differentiation, convolution)
- Schwartz space, tempered distributions, Fourier transform

Solving PDEs:
- Solving a basic PDE using Fourier transform and distributions
- Fundamental solutions for linear PDEs with constant coefficients

Fourier series:
- Fourier coefficients and their main properties
- convolution theorem
- L^2 and pointwise convergence

Assessment

Continuous assessment 30%
Written examination 70%

- **MTH4322 Topological Data Analysis (Sem 1) [offered in 2025/26]**

Pre-requisite: MTH2011 Linear Algebra

Lecturer:

Introduction

Topological data analysis (TDA) stands for a collection of powerful mathematical and computational tools that can quantify shape and structure in data in order to answer questions from the data's domain.

The aim of this module is to introduce the main mathematical tools and techniques needed to understand and apply modern methods from topological data analysis.

Content

- Simplicial complexes
- PL functions
- Simplicial homology
- Filtrations and barcodes
- Matrix reduction
- The Mapper Algorithm
- Learning with topological descriptors
- Statistics with topological descriptors

Assessment

Continuous assessment 25%
Written examination 75%

- **MTH4323 Geometry of Optimisation (Sem 1) [offered in 2024/25]**

Pre-requisite: MTH2011 Linear Algebra

Lecturers: Dr T Huettemann, Dr S Moutari

Introduction

Polytopes are geometric objects with "flat faces". A very familiar example is the (three-dimensional) cube which has 8 vertices, 12 edges and 6 two-dimensional faces. Other familiar examples are the tetrahedron (a pyramid on a triangle) and an Egyptian pyramid (with a square base). Polytopes have a very rich and interesting combinatorial structure; a well-known example is Euler's formula, which asserts that (for a three-dimensional polytope) the number of vertices minus the number of edges plus the number of two-dimensional faces always equals 2. While much more is known about these beautiful objects, polytope theory remains an area of active research!

Given a specific polytope it is quite "obvious" what its faces should be: We can easily identify the vertices, edges and so on in any given example. The general, abstract definition of a face has connections to (linear) optimisation: Faces are in fact the set of points where a certain (linear) function takes its largest value on the polytope. We may think of the function representing "profit", and maximising profit (under certain constraints) is a typical optimisation problem. The constraints define the polytope; typical constraints are, for example, that quantities must not be negative (you can't own a negative number of things!), or that the maximal amount of available funding is fixed (so one cannot invest more than that amount). For example, asking for x and y to be non-negative, and $x + y$ to be at most 10, defines a triangular region in the plane with vertices at the origin, at (10,0) and at (0,10).

In this module, you will learn both the foundational theory of polytopes, leading to a geometric understanding of linear optimisation problems and their solutions, and practical methods to solve optimisation problems, most notably the "simplex algorithm".

Content

- Functionals on n-dimensional space, linear equations and inequalities; hyperplanes; half-spaces
- Convex polytopes; faces
- Specific examples: e.g., travelling salesman polytope, matching polytope
- Linear optimisation problems; geometric interpretation; graphical solutions
- Simplex algorithm
- LP duality
- Further topics in optimisation, e.g., integer programming, ellipsoid method

Assessment

Exam 75%
Coursework 25%

- **MTH4331 Quantum Fields (Sem 2) [SUSPENDED]**

Pre-requisite: MTH3032 Quantum Theory and MTH3032 Quantum Theory, or PHY3001 Quantum Mechanics and Relativity

Lecturer:

Introduction

Quantum field theory is the quantum theory of fields, just like quantum mechanics describes quantum particles. The term “field” might refer to various entities: a field of a classical field theory, such as electromagnetism, the wave function of a particle in quantum mechanics, the smooth approximation to some property in a solid, such as the displacement of atoms in a lattice, some function of space and time!

This module presents an introduction of the ideas of quantum field theory and of simple applications thereof, including quantization of classical field theories, second quantization of bosons and fermions, solving simple models using second quantization, path integral approach to quantum mechanics and its relation to classical action principles, field integrals for bosons and fermions, the relationship between path integral methods and second quantization, solving many-body quantum problems with mean-field theory, and applications of field theoretic methods to models of magnetism.

Content

- Review of foundations of quantum theory
- Introduction to relativistic Lagrangian field theory
- The role of symmetries and associated conservation laws (Noether’s theorem)
- The Klein-Gordon field and its second quantisation
- The Dirac field and equation including the spin statistic theorem
- Covariant electromagnetic field quantisation
- S-matrix and Wick’s theorem
- Basic concepts of quantum electrodynamics (Feynman’s diagrams).

Assessment

Assignment 20%
Presentation 20%
Exam 60%

- **MTH4332 Statistical Mechanics (Sem 2) [offered in 2025/26]**

Pre-requisite: MTH3032 Quantum Theory or PHY3001 Quantum Mechanics and Relativity

Lecturers: Dr T Todorov and Dr G Tribello

Introduction

Statistical mechanics is a theoretical framework in physics that can be used to rationalise the behaviour of physical systems based on an understanding of the individual atoms of which they are composed. This module provides an introduction to the subject that combines a theoretical section with a practical component. You will learn the relevant theory through a set of lectures, and apply the knowledge by writing computer programs to simulate systems of spins and atoms. In the final assignment you will then engage with the cutting-edge research literature for this field.

Content

<u>Lectures:</u> *The three principal ensembles:* entropy; temperature, pressure and chemical potential; ensemble entropy; canonical ensemble; grand-canonical ensemble; distinguishable and indistinguishable particles; from total energy and particle number to the occupations of single-particle states; ideal Fermi gas; ideal Bose gas; classical limit. *Phase transitions:* paramagnetism; Ising model. *Phonons and photons:* quantum harmonic oscillator; dynamical response matrix and phonons; internal energy and heat capacity; Debye model; blackbody radiation.

<u>Computer practicals:</u> The central object in statistical mechanics is the partition function. To evaluate this function, you have to evaluate a high-dimensional integral. Most undergraduate courses on this topic focus on exceptional cases that are idealized in a way that ensures these integrals can be determined analytically. Consequently, these courses neglect the numerical techniques for determining ensemble averages, which are the workhorses for cutting-edge research in this field because it is difficult to assess student understanding of these ideas through examinations. In the practical, computer-based parts of this module, you will be introduced to the Monte Carlo and molecular dynamics algorithms and shown how these techniques can be used to estimate physical observables. This part of the course is more experiential and discursive and is designed in a way that we hope provides you with an opportunity to engage more deeply with the ideas introduced in the lectures.

Assessment

Exam 45%
Coursework 55%

- **PMA4001 Project (Full year)**

Pre-requisite: This project is a compulsory component of the MMath/MSci pathway in Pure Mathematics. There is no specific pre-requisite for this module, but the student will need enough Level 3 background in Pure Mathematics to undertake an extended project at this level in some area of Pure Mathematics for which supervision can be offered.

Co-ordinator: Dr Y-F Lin

Introduction

This is an extended project designed to test the student's ability to work independently at a high level for a prolonged period of time with a restricted amount of supervision. This will give a taste of the kind of work expected of a mathematician in the commercial or academic world, unlike the relatively short bursts of work expected in most undergraduate modules. It will also provide an opportunity to develop those transferable skills that are sought by employers, including IT (both word-processing and data-base access), presentational and personal ones.

Content

The project takes place during the two terms of Level 4. It will normally involve study and exposition of a piece of mathematical work beyond the normal undergraduate syllabus and which will probably not be available in easily assimilated form. Originality of exposition will be expected, but not necessarily much in the way of original results. The main part of the assessment will consist of a word-processed report, but 20% of the marks for the project are awarded for an oral presentation of the work which will take place before or after Easter, depending on the academic calendar. As preparation for this assessed oral presentation, the student will be expected to give one oral progress report to a small group of staff and any other students undertaking this module. Constructive advice on this presentation will be provided.

Students intending to take this module should seek advice and think about their choice of project during the summer. The selection of a project should be finalized no later than the start of the academic year, and it would be helpful to all involved if students actually did this even earlier.

Assessment

Dissertation 80%
Presentation 20%

- **SOR4001 Project (Full year)**

Pre-requisite: This two-semester-long double module is only available to students on the MMath Mathematics and Statistics & Operational Research pathway.

Co-ordinator: Dr K Cairns, Dr H Mitchell

Introduction

In the MMath/MSci Statistics Project, students will complete a substantial investigation of a statistical or operational research problem over the course of two semesters. This project will incorporate a review of relevant literature, use of statistical software packages and, when necessary, the construction of computer programs to solve specific stages of the problem. The research undertaken will be presented in the form of a technical report and a sequence of oral presentations culminating in a 30-minute assessed presentation.

Content

The mathematical contents of the project will depend on the nature of the research problem.

Assessment

Dissertation 80%
Presentation 20%

- **SOR4008 - Bayesian Statistics (Sem 1)**

Pre-requisite: SOR2002 Statistical Inference or SOR3012 Stochastic Processes and Risk

Lecturer: Dr H Mitchell

Introduction

Within the last decade, the demand for Bayesian statistics has grown dramatically in both industry and research. Bayesian estimation is a collection of inferential methods based on the use of Bayes' Theorem, providing the means of incorporating prior beliefs when estimating unknown parameters.

Content

This module will cover an introduction to Bayesian analysis, models for both discrete and continuous data, the concept of alternative priors and non-conjugate models and estimation utilising Markov Chain Monte Carlo. The theory will be applied to common statistical models and include model checking procedures such as the use of trace plots.

Assessment

Exam 60%
Coursework 30%
Presentation 10%

- **SOR4007 Survival Analysis (Sem 2)**

Pre-requisite: SOR3004 Linear Models

Lecturer: Dr L McFetridge

Introduction

Survival analysis is an important tool for research in medicine and epidemiology. It is that part of statistics that deals with time-to-event data. For example, in a clinical study the data might consist of the post-treatment survival times of patients with hypernephroma (i.e., a malignant tumour of the kidney). Survival analysis might address questions such as:

- How does the patient's survival time depend on her age at treatment?
- What is the effect of kidney removal on the survival times of patients compared with others who are treated just with chemotherapy?
- Is the size of the tumour an equally good predictor of survival for patients under 60 years of age as for the over 60s?

The module introduces the student to the special features of survival data such as censoring (e.g. where a patient is lost to follow up but is known to have survived to a particular time) and positive skew in the distribution of survival times. Fundamental concepts of survival analysis will be introduced including the survivor function, the hazard function and the hazard ratio. The course will build from some elementary nonparametric techniques such as the Kaplan-Meier estimate of the survival curve to the Cox proportional hazards model - one of the most flexible and widely used tools for the analysis of survival data.

An important element of this module will be putting the theory into practice using statistical software packages. Computer practical sheets and online quizzes will give students the opportunity to analyse real world survival datasets and gain further insights into the kind of interpretations and knowledge that can be gained from this type of analysis.

Content

Survival data, survivor and hazard functions. Nonparametric method: estimating median and percentile survival and confidence intervals. Comparing two groups of survival data, the log-rank and Wilcoxon tests. Comparison of k-groups. The Cox proportional hazard model, baseline hazard, hazard ratio, including variates and factors, maximum likelihood, treatment of ties. Confidence intervals for the Cox model regression parameters and hypothesis testing. Estimating the baseline hazard. Model building, Wald tests, likelihood ratio tests and nested models.

Assessment

Exam 60%
Coursework 30%
Presentation 10%